

**NORTH MAHARASHTRA UNIVERSITY
JALGAON**



SYLLABUS

FOR

M.Sc. STATISTICS (Semester I and II)

**WITH EFFECT FROM ACADEMIC
YEAR 2002-2003**

NORTH MAHARASHTRA UNIVERSITY, JALGAON
Syllabus for M.Sc. (Statistics)

With effect from Academic Year 2002-2003

Syllabus Structure

Semester-I

Course No.	Title of the Course	Contact hours / week			Marks Distributions for Examinations					
					Internal		External		Total	
		Th(L)	Pr	Total	Th	Pr	Th	Pr	Th	Pr
ST-101	Real Analysis	04	--	04	40	--	60	--	100	--
ST-102	Linear Algebra	04	--	04	40	--	60	--	100	--
ST-103	Sample Surveys and Statistics for National Development	04	--	04	40	--	60	--	100	--
ST-104	Distribution Theory	04	--	04	40	--	60	--	100	--
ST-105	Computer Programming in C++ and Numerical Methods	04	--	04	40	--	60	--	100	--
ST-106	Practicals-I	--	06	06	--	40	--	60	--	100

Semester-II

Course No.	Title of the Course	Contact hours / week			Marks Distributions for Examinations					
					Internal		External		Total	
		Th(L)	Pr	Total	Th	Pr	Th	Pr	Th	Pr
ST-201	Probability Theory	04	--	04	40	--	60	--	100	--
ST-202	Stochastic Processes	04	--	04	40	--	60	--	100	--
ST-203	Multivariate Analysis	04	--	04	40	--	60	--	100	--
ST-204	Parametric Inference	04	--	04	40	--	60	--	100	--
ST-205	Linear Models and Regression Analysis	04	--	04	40	--	60	--	100	--
ST-206	Practicals-II	--	06	06	--	40	--	60	--	100

Th: Theory

Pr: Practicals

L: Lectures

M: Marks

General Instructions to Teachers and Paper Setters/ Examiner

1. Each Theory Course required 60 lectures each of one hour.
2. Each Practical Course required 90 laboratory periods each of one hour.
3. Numbers of Lectures/periods to be devoted for each topic and minimum number of marks to be allotted out of 60 for main topics are mentioned in parentheses in the detailed syllabi.
4. Teacher should follow syllabus as well as time schedule given in the syllabus for all topics. Variation up to 4/5 hours (more or less) may be acceptable.
5. Each external examination theory question paper should contain 8 questions of 12 marks each. Student will be asked to answer 5 questions out of 8 questions. Each external examination practical question paper should contain 6 questions each of 15 marks and student will be asked to answer any 4 questions. Paper Setter may consider sub parts such as ((a), (b), (c)...)) in each question.
6. Question paper should generally be uniformly distributed over the syllabus.

Examination Pattern:

Number of Internal Tests and Time duration:

Concern Teacher in consultation with Head of the Department may conduct 2 or 3 tests of 40 marks (2 Hrs.) for Internal Examination of all Theory and Practical courses. Head of the Department well in advance will declare detailed Timetable.

External Examination:

University will conduct external examinations at the end of each semester. Each course will have examination of 60 marks (3 Hrs). Practical examination will be conducted before Theory examination. Timetable for external examinations will be declared by University office well in advance.

Standard of Passing:

To pass any course, the candidate has to secure at least 40% marks in the internal as well as in the external examinations.

The candidate failed in Internal or External or in both examinations shall have to appear for subsequent Internal or External or both Examinations respectively for that course.

The student having the backlog of any course(s) from first year of M.Sc. can be admitted to second year of M.Sc.

NORTH MAHARASHTRA UNIVERSITY, JALGAON

Syllabus for M.Sc. (Statistics)

With effect from Academic Year 2002-2003

Detailed Syllabi: First Year

ST-101: REAL ANALYSIS

- **The Real Number System: (8 Marks)**
 - Introduction, The field axioms, The order axioms, Geometric representation of real numbers, Intervals, Integers, The unique factorization theorem for integers. (2L)
 - Rational numbers, Irrational numbers, Upper bounds, Lower bounds, Least upper bound (supremum), Greatest lower bound (infimum) of the sets of real numbers. (2L)
 - The completeness axiom, Some properties of the supremum and infimum, Archimedean property of the real number system. (2L)
 - Rational numbers with finite decimal representation, Finite decimal approximations to real numbers, Infinite decimal representation of real numbers. (1L)
 - Absolute values and the triangle inequality, The Cauchy-Schwarz inequality, Plus and minus infinity and the extended real number system R^* . (1L)
- **Basic Notions of Set Theory: (6 Marks)**
 - Ordered pairs, Cartesian product of two sets, Relations and functions. (1L)
 - Further terminology concerning functions, One-to-one functions and inverses, Composite functions, Sequences. (2L)
 - Similar (equinumerous) sets, Finite and infinite sets, Countable and uncountable sets, Uncountability of real number system. (2L)
 - Set algebra, countable collections of countable sets and related results. (2L)
- **Elements of Point Set Topology: (5 Marks)**
 - Introduction to n -dimensional Euclidean space, Open and closed intervals (rectangles), Open and closed sets on the real line, limit points of a set, Compact set. (2L)
 - The Bolzano-Weierstrass theorem and Heine-Borel theorem for real line R (without proof). (2L)
- **Sequences and Series of Real Numbers: (15 Marks)**
 - Introduction and examples of sequences of real number. (1L)
 - Convergence of sequences, limit of a sequence, limit superior and limit inferior of a real-valued sequences, Monotone sequences of real numbers. (3L)
 - Cauchy sequences and related results. (2L)
 - Infinite series, Alternating series. (1L)
 - Convergence of Series, Absolute and conditional convergence. (1L)
 - Test for convergence of series with positive terms (Comparison test and limit comparison test). (1L)
 - The geometric series. (1L)

- The integral test. (1L)
- The big oh and little oh notation. (1L)
- The Ratio test and Root test, Abel's test. (2L)
- **Limit and Continuity: (6 Marks)**
 - Limits of functions. (1L)
 - Continuous functions. (1L)
 - Uniform continuity. (1L)
 - Discontinuities. (1L)
 - Continuity and compactness. (2L)
 - Monotone function and discontinuities. (1L)
- **Sequences of Functions: (6 Marks)**
 - Introduction and examples of sequences of real-valued functions. (1L)
 - Pointwise convergence of sequences of functions. (1L)
 - Definition of uniform convergence, Uniform convergence and continuity. (2L)
 - Power series and radius of convergence. (1L)
- **Differentiation and functions of several variables. (6 Marks)**
 - The Derivative of a Real Function. (1L)
 - Maxima-minima of function, Mean value theorems. (2L)
 - The Continuity and Derivatives. (1L)
 - Derivatives of higher order, Taylor's theorem (without proof). (1L)
 - Functions of several variables, constrained maxima-minima functions. (2L)
- **Integrals: (8 Marks)**
 - Multiple integrals and their evaluation by repeated integration. (2L)
 - Change of variables in multiple integration. (2L)
 - Uniform convergence in improper integrals. (2L)
 - Differentiation under the sign of integral Leibnitz rule. (2L)

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- Apostol, T.M. (1985). *Mathematical Analysis*, (Narosa, Indian Ed.)
 Courant, R. and John, F. (1965). *Introduction to Calculus and Analysis*, (Wiley).
 Miller, K.S. (1957). *Advanced Real Calculus*, (Harper, New York).
 Rudin, Walter (1976). *Principles of Mathematical Analysis*, (McGraw Hill).

ST-102: LINEAR ALGEBRA

- **Preliminaries: (2 Marks)**
 - Binary operations, Groups, Polynomials. (2L)
- **Vector Spaces (VS) : (12 Marks)**
 - Definition of VS, Subspaces, Linear span of a set, examples of VS over real and complex fields. (3L)

- Linear span of a set, Span of a set as a smallest subspace containing the set, Generating set of a VS, Results on span of a set. (2L)
- Intersection and union of sub spaces, Completion theorem. (2L)
- Linear dependence and linear independence of set of vectors, Necessary and sufficient condition for linear dependence of set of vectors. (2L)
- Steinitz exchange theorem, Maximal linearly independent set, minimal generating sets. (2L)
- Basis of VS, Dimension, Extension of linearly independent set to a basis (algorithm and theorem), relation between dimensions of subspaces; one of which is subset of other. (4L)
- Sum of 2 sets, modular law. (1L)

• **Algebra of Matrices: (12 Marks)**

- Linear transformations and matrices, Addition, Scalar multiple and composition of linear transformation, The corresponding operations on matrices, Elementary properties of matrix operations, Upper and lower triangular matrices, Trace of a matrix and related results. (3L)
- Row and column spaces, Rank of a matrix, Left inverse, Right inverse and inverse of a matrix, properties of inverse, Upper bound for rank of product of matrices, Rank cancellation laws. (3L)
- Nullity of matrix, null space of a matrix, relation of rank of null space of a matrix with rank of a matrix. (2L)
- Lower bound for rank of product of 2 matrices, Rank of sum of matrices. (1L)
- Partitioned matrix, Elementary matrix, Determinant of a matrix, Its elementary properties, Determinant and inverse of partitioned matrix, Kronecker product. (3L)

• **Linear Equations of Systems of Equations: (8 Marks)**

- Consistent and inconsistent system of equations, Homogeneous systems and existence of nontrivial solution for it, General linear systems, Solution of systems of equations. (3L)
- Generalized inverse of a matrix and its properties, Moore-Penrose generalized inverse, Solution of systems of equations. (3L)

• **Inner Product and Orthogonality: (10 Marks)**

- VS with inner product, Normed vector spaces, Cauchy-Schwarz inequality, Orthogonality and linear independence. (2L)
- Orthonormal basis, Expression of any vector in VS as a linear combination of elements of orthonormal basis. (2L)
- Gram-Schmidt orthogonalization process, Extension of any orthogonal set to orthonormal basis of VS, Examples. (2L)
- Orthogonal and unitary matrices, and their properties. (2L)

• **Eigen Values: (8 Marks)**

- Characteristic polynomial and characteristic equation of a matrix, Characteristic roots, their properties. (2L)
- Eigen values and eigen vectors, Eigenspaces, Geometric and algebraic multiplicity of an eigen value, Relation between the 2 multiplicities, Simple and regular eigen values, Properties of eigen values. (3L)
- Cayley-Hamilton theorem and minimal polynomial, Singular values and singular vectors. (2L)

- Spectral decomposition of real symmetric matrix, singular value decomposition, Jordan decomposition. (2L)
- **Quadratic Forms (QFs): (8 Marks)**
 - Real QFs, Classification, Rank and signature, reduction of any QF to diagonal form. (2L)
 - Definiteness of a matrix, equivalence of nonnegative definite matrix and variance-covariance matrix, Simultaneous reduction of 2 QFs. (3L)
 - Extrema of QFs, Maxima and Minima of ratio of 2 QFs. (2L)

REFERENCES

- Graybill, F.A.(1983). *Matrices with Applications in Statistics* (2nd Ed. Wadsworth)
- Rao, A.R.and Bhimasankaram, P.(1992):*Linear Algebra*. (Tata McGraw Hill Publishing Company Ltd).
- Rao, C.R.(1973). *Linear Statistical Inference and its Applications*. (2nd ed.John Wiley and Sons, Inc).
- Searle, S. R.(1982). *Matrix Algebra Useful for Statistics*. (John Wiley and Sons.Inc).

ADDITIONAL REFERENCES

- Bellman, R.(1970). *Introduction to Matrix Analysis*. (2nd ed. McGraw Hill).
- Eiswas, S.(1984). *Topics in Algebra of Matrices*. (Academic Publications).
- Hadley, G.(1987). *Linear Algebra*. (Narosa Publishing House).
- Halmos, P.R.(1958) *Finite-dimensional Vector Spaces*, (2nd ed.D.Van Nostrand Company, Inc).
- Hoffman, K.and Kunze, R(1971): *Linear Algebra*. (2nd Ed.Prentice Hall, Inc.)
- Fao, C.R. and Mitra, S.K.(1971): *Generalized Inverse of Matrices and its Applications*.(John Wiley and Sons. Inc).

ST-103: SAMPLE SURVEYS AND STATISTICS FOR NATIONAL DEVELOPMENT

- **Sample Surveys:**
 - **Review of basic methods of sample selection from finite population. (10 Marks)**
 - Simple random sampling with replacement, Simple random sampling without replacement, Stratified sampling, Systematic sampling and related results on estimation of population mean/total. (7L)
 - Allocation problem in stratified sampling. (3L)
 - **Unequal Probability Sampling Designs: (8 Marks)**
 - PPSWR, PPSWOR methods (including Lahiri's scheme) and related estimators of a finite population mean (Jensen-Horwitz and Desraj estimators for a general sample size and Murthy's estimator for a sample of size 2). (7L)

- Inclusion probabilities, Horwitz-Thompson estimator and its properties. (2L)
- Midzuno Sampling design. (2L)
- Use of supplementary information for estimation, Ratio and Regression estimators based on SRSWOR method of sampling. Their properties and MSE's. (5 Marks) (4L)
- The Jackknife technique. (2 Marks, 1L)
- Cluster sampling, Estimator of population mean and its properties. (4 Marks, 3L)
- Two-stage sampling with equal number of second stage units. (2 Marks, 2L)
- Double sampling and its uses in ratio and regression estimation. (3 Marks, 3L)
- Randomized response technique, Warner's model; related and unrelated questionnaire methods. (4 Marks, 4L)
- **Statistics for National Development:**
 - **Economic Development: (8 Marks)**
 - Growth in per capita income and distributive justice. (2L)
 - Indices of development. (2L)
 - Human Development indexes. (1L)
 - Estimation of national income-product approach, income approach and expenditure approach. (4L)
 - Population growth in developing and developed countries. Population projection using Leslie matrix, Labour force projection. (4 Marks, 4L)
 - Measuring inequality in incomes, Gini coefficient, Theil's measure. (3 Marks, 2L)
 - **Poverty measurement: (8 Marks)**
 - Different issues related to poverty. (3L)
 - Measures of incidence and intensity. (2L)
 - Combined measures e.g. Indices due to Kakwani, Sen etc. (2L)

REFERENCES

Sampling Methods:

- Chaudhari, A. and Mukerjee, R (1988): Randomized Response: Theory and Techniques (New York, Marcel Dekker Inc).
- Cochran, W.G. (1977): Sampling Techniques. (3rd Edition Wiley)
- Dés Raj and Chandak (1998): Sampling Theory (Narosa).
- Murthy, M.N. (1977): Sampling Theory & Methods (Statistical Publishing Society, Calcutta).
- Sukhtme et. El. (1984): Sampling Theory of Surveys with Applications (Iowa State University Press & IAIS).
- Singh, D. and Chaudhary, F.S. (1986): Theory and Analysis of Sample Survey Designs (New Age International Publishers).

Statistics for National Development:

- CSO (1980): National Accounts Statistics- Sources and Health.
- Keyfitz, N. (1977): Applied Mathematical Demography, Springer Verlag.
- UNESCO: Principles for Vital Statistics Systems. Series M-12.
- Sen, A. (1997): Poverty and Inequality.

ST-104: DISTRIBUTION THEORY

- **Brief review of basic distribution theory: (5 Marks)**
 - Random experiment and its sample space, events. (1L)
 - Probability axioms. (1L)
 - Random variables, Discrete random variables, Continuous random variables. (1L)
 - P.d.f., p.m.f., c.d.f. of random variables. (1L)
 - M.g.f., p.g.f., c.g.f., characteristic function of random variables. (1L)
 - Moments: raw moments, Central moments, Factorial moments. (1L)

- **Standard discrete and continuous distributions: (8 Marks)**
 - Bernoulli, Binomial, Geometric, Negative Binomial, Poisson, Hypergeometric distributions. (2L)
 - Exponential, Normal, Gamma, Beta, Uniform, Chi-square, Lognormal, Weibull, Cauchy distributions. (3L)
 - M.g.f, p.g.f., c.g.f., characteristic function, Moments of above distributions. (2L)
 - Properties of above distributions. (2L)

- **Joint, Marginal and Conditional distributions: (10 Marks)**
 - Concept of random vectors, Joint, Marginal and conditional distributions Variance-covariance matrix. (2L)
 - Joint p.m.f. of discrete random variables, Joint p.d.f. of continuous random variables. (2L)
 - Marginal and conditional density using joint density. (1L)
 - Conditional expectation and variance. (1L)
 - Independence of random variables. (1L)
 - Bivariate normal distribution: Joint p.d.f. Marginal p.d.f.s, Conditional p.d.f., Joint m.g.f., Some properties. (2L)
 - Bivariate exponential distribution: joint p.d.f., Marginal p.d.f.s, properties. (1L)
 - Multivariate normal distribution: joint p.d.f., Marginal p.d.f., Conditional p.d.f., Joint m.g.f. (2L)
 - Multinomial distribution: joint p.m.f., Marginal p.m.f., Conditional p.m.f., Joint m.g.f. (2L)

- **Functions of random variables and their distributions: (10 Marks)**
 - Function of random variables. (1L)
 - Joint density of functions of random variables using jacobian of transformation. (2L)
 - Convolution of random variables. (1L)

- **Compound, Truncated and Mixture Distributions: (3 Marks)**
 - Concept, applications, examples and problems. (3L)

- **Correlation: (3 Marks)**
 - Multiple and Partial Correlation. (2L)

- **Regression: (3 Marks)**
 - Linear and multiple regression, Regression Function, Best linear regression function. (3L)

• **Sampling Distributions: (6 Marks)**

- Introduction, Sampling distribution of statistics from univariate normal random samples. (2L)
- Non-central Chi-square, t and F- distributions and their properties. (5L)

• **Quadratic forms under Normality: (6 Marks)**

- Distribution of linear and quadratic forms in i.i.d. Standard normal variables (Technique based on m.g.f.). (2L)
- Independence of two linear forms, Independence of two quadratic forms and independence of linear form and quadratic form. (2L)
- Cochran's theorem. (2L)

• **Order Statistics: (6 Marks)**

- Order statistics, Their distributions and properties. (2L)
- Joint and marginal distributions of order statistics. (2L)
- Distribution of function of order statistics. (2L)
- Extreme values and their asymptotic distributions (statement only) with applications. (2L)

REFERENCES

- Dudewicz, E.J. and Mishra, S.N. (1988): Modern Mathematical Statistics (Wiley, Int'l Students' Eds).
- Hogg, R.V. and Craig, A.T. (1970): Introduction to Mathematical Statistics (3rd Ed. The Macmillan Co. Collier-Macmillan Ltd., London).
- Rao, C.R. (1973): Linear Statistical Inference and Its Applications (2/e, Wiley Eastern).
- Rohatgi, V.K. (1984): An Introduction to Probability Theory and Mathematical Statistics (Wiley Eastern).

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- Bitman, J. (1993): Probability (Narosa Publishing House).
- Johnson, S. and Kotz, (1972): Distributions in Statistics (Vol. I, II and III, Houghton and Mifflin).
- Cramer H. (1946): Mathematical Methods of Statistics (Princeton).

ST-105: COMPUTER PROGRAMMING IN C++ AND NUMERICAL METHODS

Computer Programming in C++ (40 Marks)

- Principles of Object-Oriented Programming. (3L)
- Beginning with C++. (3L)
- Tokens, Expressions and Control Structures. (6L)
- Functions in C++. (3L)
- Classes and Objects. (5L)

- Constructors and Destructors. (5L)
- Operator Overloading and Type Conversions. (5L)
- Inheritance: Extending Classes. (3L)
- Pointers, Virtual Functions and Polymorphism. (3L)
- Managing Console I/O Operations. (3L)
- Working With Files (including linking to databases). (4L)

Numerical Methods: (20 Marks)

- **Errors in Numerical Calculations:**
Introduction, Errors and their Analysis. A general error formula, error in series approximation. (2L)
- **Iterative methods:**
Introduction, the method of successive bisection, Newton-Raphson method. (2L)
- Interpolation (1L)
- **Solution of Simultaneous Algebraic Equations:**
Introduction, Matrix Inversion Method, The Gauss elimination method, Pivoting, The Gauss-Seidel iterative method, The Eigen value Problem. (3L)
- **Numerical Integration:**
Introduction, Simpson's 1/3 rule, Trapezoidal Rule, Quadrature Rule, Simpson's 3/8 Rule, Errors in integration formulae. Monte Carlo integration. (5L)

REFERENCES

- E. Balagurusamy (2001): Object-Oriented Programming with C++. (2nd Ed. Tata Mcgraw Hill).
- Gottfried: Programming in C++ (Schaum's Outline Series).
- K.R.Venugopal, Rajkumar, J.Ravishankar: Mastering C++.
- V.Rajaraman (1993): Computer Oriented Numerical Methods. (3rd Ed. Prentice-Hall)
- W.H.Press, S.A.Teukolsky, W.T. Vetterling and B.P.Flannery (1993): Numerical Recipes in C, (2ndEd.Cambridge University Press).
- K.A.Thisted (1988) : Elements of Statistical Computing (Chapman and Hall).

ST-106: PRACTICALS-I

- A. **Introduction to MS-DOS, WINDOWS-98, Introduction of Software Packages (MINTAB, MATLAB, SPSS, MS-EXCEL) (5 Marks) (12 Hrs.: First 2 weeks)**

(Introduction can be done through following simple practicals)

1. Classification, Tabulation and Frequency Tables.
2. Bar Graphs, Histogram.
3. Stem-and- Leaf Plots, Box Plots.
4. Summary Statistics.
5. Two-way Tables and Plots.
6. Scatter diagram Correlation Coefficient.

B. Practicals based on Computer Programming C++ and Numerical Methods. (40 Hrs, 25 Marks)

1. Writing programs to calculate different statistical terms based on n observations.
2. Write a class to represent a vector. Include member functions to perform the following tasks:
 - To create the vector.
 - To modify the value of a given element.
 - To multiply by a scalar value.
3. Program to find product of two matrices.
4. Programs to compute the c.d.f.'s of standard probability distributions.
5. Drawing random samples from discrete, continuous and mixture distributions.
6. Programs based on the numerical methods taught in the course ST-105.
7. Some other programs in C++ useful in Probability, Statistics and Data Representation & Analysis.

C. Practicals based on the Sampling Theory and Statistics for National Development. (Using software packages) (26 Hours, 20 Marks)

1. **Model Sampling (3L)**
 - Drawing simple random samples from a given population.
 - Estimating the population mean using the drawn sample.
 - Estimating the variance of the estimator used above using the drawn sample.
2. **Stratified Random Sampling (4L)**
 - Various kinds of allocation and estimation of population mean with S.E.
 - Post stratification.
3. **Using Auxiliary Information (4L)**
 - Ratio method of estimation
 - Regression method of estimation.
4. **H-T estimator and PPS, π PS designs (2L)**
5. **Double Sampling. (2L)**
Cost consideration Comparison of different allocations through variance of the estimator keeping cost fixed.
6. **Systematic Sampling (2L)**
7. **Cluster Sampling (2L)**
8. **Two stage sampling (2L)**
9. **Randomized Response Technique (2L)**
10. **Practical based on Estimation of national income, Poverty measurement. (3L)**

D. Practicals based on Linear Algebra. (Using software packages)(8Hours,6Marks)

1. **Computation of inverse of a given matrix. (2L)**
 - Natural inverse.
 - G-inverse
 - MP-inverse
2. **Solving system of Linear Equations. (2L)**
3. **Computing higher order powers of a given matrix using spectral decomposition. (1L)**

4. Diagonalization and triangulation of a given matrix. (1L)
 5. Gram-Schmidt orthonormalization process. (2L)
- E Practicals based on Distribution Theory. (Using software packages)(4 Marks)**
1. Generating random samples from discrete, continuous and mixture distributions (2L)
 2. Fitting of standard distributions and tests for goodness of fit. (2L)

ST-201: PROBABILITY THEORY

- **Sets and Classes of Events: (6 Marks)**
 - Random Experiment, Sample space and events. (1L)
 - Algebra of sets. (1L)
 - Sequence of sets, limsup and liminf of sequence of sets. (2L)
 - Classes of sets, Sigma-fields (σ -fields), Minimal fields, Minimal σ -field, Partition. (3L)
 - Borel fields in R^1 and R^k , Monotone field. (2L)
- **Random variables: (6 Marks)**
 - Point function and set function, Inverse function. (2L)
 - Measurable function, Borel function, Induced σ -field, Function of a function, Borel function of measurable function. (2L)
 - Real and vector-valued random variable. (2L)
 - σ -field induced by a sequence of random variables. (1L)
 - Limits of Random variable. (2L)
- **Measure and Probability Measure: (6 Marks)**
 - Measure (Definition and simple properties). (1L)
 - Probability measure, Properties of a measure. (1L)
 - Probability space (finite, countable) Continuity of a probability measure. (1L)
 - Extension of probability measure, Caratheodory Extension theorem (without proof). (1L)
 - Probability space induced by r.v. X, Distribution of Borel functions of r.v.s. (1L)
 - Other measures: Generalized Probability measure, Conditional Probability measure, Counting measure, Lebesgue measure. (2L)
- **Distribution Functions: (6 Marks)**
 - Distribution function of a r.v. and its properties. (1L)
 - Jordan decomposition theorem, Mixture of distribution functions. (2L)
 - Distribution functions of vector valued r.v.s. (1L)
 - Empirical distribution functions. (1L)
- **Expectation and Moments: (8 Marks)**
 - Integration of measurable function with respect to a measure. (1L)
 - Expectation of a r.v. (Definition for simple, Nonnegative and arbitrary r.v.), Properties of expectation, Expectation of Complex r.v. (3L)
 - Moments, Moment generating function. (1L)

- Moment inequalities: C_r -inequality, Hölder inequality, Schwarz's inequality, Minkowski's inequality, Jensen's inequality, Liapounov's inequality, Basic inequality, Markov inequality, Chebyshev's inequality. (3L)
- **Convergence of Sequence of Random variables: (8 Marks)**
 - Convergence in distribution, Convergence in probability, Almost sure convergence and convergence in quadratic mean and their inter-relations. (5L)
 - Monotone convergence theorem, Fatou's Lemma, Dominated convergence theorem. (3L)
- **Characteristic function: (4 Marks)**
 - Definition and simple properties, Some inequalities. (2L)
 - Uniqueness theorem (statement only), Levy's continuity theorem (Statement only). (1L)
- **Independence: (5 Marks)**

Independence of two events, Independence of $n > 2$ events, sequence of independent events, independent classes of events, independence of r.v.s, Borel zero-one law. (4L)
- **Law of large numbers: (5 Marks)**
 - Weak laws of large numbers (WLLN), Khintchine's WLLN, Kolmogorov's strong law of large number (Statement only) and their applications. (4L)
- **Central limit theorem (CLT): (6 Marks)**
 - CLT for a sequence of independent r.v.s. under Lindeberg's condition, CLT for i.i.d. r.v.s. and its applications. (3L)

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- Ash, Robert. (1972): Real Analysis and Probability (Academic Press).
- Bhat, B.R. (1999): Modern Probability Theory (3rd Ed. New Age International (P) Ltd. Publication)
- Billingsley, P. (1986): Probability and Measure (Wiley).
- Dudley, R.M. (1989): Real Analysis and Probability (Wadsworth and Brook/Cole).
- Kingman, J.F.C. and Taylor, S.J. (1966): Introduction to Measure and Probability. (Cambridge University Press).
- Basu, A.K. (1999): Measure Theory and Probability (Prentice Hall of India).

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- Feller, W. (1969): Introduction to Probability and its applications vol. II (Wiley Eastern Ltd.)
- Loeve, M. (1978): Probability Theory (4th Ed. Springer Verlag).
- Gnedenko, B.V. (1988): Probability Theory (Mir. Pub.).

ST-202: STOCHASTIC PROCESSES

- **Introduction: Stochastic Processes, Markov chains (16 Marks)**
 - Review of conditional probability and Expectation with problems. (2L)
 - Introduction to Stochastic Processes, Classification of Stochastic Processes according to state space and time domain. (1L)
 - Finite and countable state space Markov chains (Definitions and examples). (2L)
 - Chapman-Kolmogorov equations, Calculation of n-step transition probability and its limit. (2L)
 - Stationary distribution of Markov chains. (2L)
 - Classification of states, Period of the state, Transient and recurrent Markov chain and related results. (4L)
 - Random walk and gambler's ruin problem. (2L)
 - First passage time and other problems with applications. (2L)
 - Applications of Markov Chains in Social, Biological and Physical Sciences. (2L)

- **Branching Process: (6 Marks)**
 - Galton-Watson branching process. (2L)
 - Probability of ultimate extinction, Distribution of population size. (2L)
 - Applications. (1L)

- **Discrete state space continuous time Markov Chain. (12 Marks)**
 - Definition and examples. (1L)
 - Markov Pure jumps processes. (2L)
 - Kolmogorov's differential equations. (2L)
 - Poisson process (Definitions, properties and applications). (3L)
 - Birth and death processes, Machine repairmen problem. (2L)
 - Wiener process as a limit of random walk. (2L)

- **Simple Queuing Systems: (6 Marks)**
 - $M|M|1$, $M|M|s$, $M|M|\infty$ queuing systems and their applications. (2L)
 - Stationary solution for $M|M|1$, $M|M|s$, $M|M|\infty$. using birth and death process approach. (2L)

- **Renewal Theory: (10 Marks)**
 - Renewal process (Definition and examples) (2L)
 - Elementary renewal theorem and its applications (2L)
 - Statement and uses of key renewal theorem (1L)
 - Renewal reward process, Regenerative Process, Semi-Markov process. (3L)
 - Age of renewal process and residual life in renewal processes and their distributions. (2L)

- **Stationary Process: (2 Marks)**
 - Weakly Stationary and strongly stationary processes. (2L)

- **Inference in Markov chains: (8 Marks)**
- Estimation of transition probabilities, estimation of functions of transition probabilities in Markov chains, Testing of order of a Markov chains, Parametric models and their goodness of fit. (8L)

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ST-203: MULTIVARIATE ANALYSIS

- **Introduction to Bivariate Distributions: (4 Marks)**
 - Bivariate normal, Bivariate Poisson, Bivariate Exponential, Bivariate binomial, Bivariate negative binomial. (3L)
- **Multivariate Normal Distribution(MVND): (16 Marks)**
 - Singular and nonsingular MVND, Mean vector and variance covariance matrix. (3L)
 - Characteristic function of MVND. (1L)
 - Additivity property of MVND. (1L)
 - Distribution of linear combinations of individual elements of a vector having MVND, Marginal distributions, Conditional distributions (2L)
 - Necessary and sufficient condition for independence of $X^{(1)}$ and $X^{(2)}$ (two components of X). (2L)
 - Central and noncentral χ^2 distribution, their characteristic function, χ distribution. (2L)
 - Distribution of quadratic forms of normal random variables. (2L)
 - Necessary and sufficient condition for a quadratic form to have χ^2 distribution, condition for independence of two quadratic forms, applications. (2L)
 - Random sampling from MVND, Unbiased and maximum likelihood estimators of parameters of MVND, their sampling distributions, independence. (3L)
 - Sample correlation coefficients, their maximum likelihood estimators (mle), Correlation matrix and its mle. (1L)

- **Wishart distribution: (10 Marks)**
 - Wishart matrix, Derivation of Wishart distribution in canonical case and in general case. (3L)
 - Characteristic function of Wishart distribution, Additive property of Wishart distribution, Moments of Wishart distribution. (3L)
 - Properties of Wishart distribution. (4L)
 - Necessary and sufficient condition for XAX' to have Wishart distribution, Its application. (2L)
- **Hotelling's T^2 and its applications: (10 Marks)**
 - Hotelling's T^2 statistic as a generalization of square of Student's statistic. (1L)
 - Derivation of Hotelling's T^2 statistic from Likelihood Ratio Test, Application of union-intersection principle to obtain Hotelling's T^2 statistic, Invariance of T^2 statistic under scale transformation. (3L)
 - Distance between two populations, Mahalanobis D^2 statistic and its relation with Hotelling's T^2 statistic. (1L)
 - Application of Hotelling's T^2 (i) in test of equality of mean vector for one or more multivariate normal population, (ii) in test of equality of components of a mean vector of MVND, (iii) in two sample problem etc. (3L)
- **Correlation and regression: (4 Marks)**
 - Sample correlation coefficient, its null and non-null distribution, Partial and multiple correlation coefficient, Their sampling distributions and maximum likelihood estimators. (2L)
 - Multiple linear regression model. (2L)
- **Discriminant Analysis: (4 Marks)**
 - Classification and discrimination procedure for discrimination between two multivariate normal populations, sample discriminant function, Probabilities of misclassification and their estimation, Optimum error rate, Test associated with discriminant function. (5L)
- **Principal Components: (4 Marks)**
 - Introduction and need, population principal components, Finding i^{th} principal component, correlation of i^{th} principal component with k^{th} element of vector \underline{X} , principal component when Σ has special structure, Sample principal components. (3L)
- **Canonical Correlation: (4 Marks)**
 - Concept of canonical correlation as generalization of multiple correlation, Geometrical interpretation and its use, Definition of canonical correlation and canonical variables, Existence of canonical variables, Canonical correlation as a maximum root of characteristic equation of a matrix, Sample canonical correlation and canonical variables. (3L)
- **Multivariate Analysis of Variance (MANOVA): (4 Marks)**
 - MANOVA for one way and two ways classified data, Wilk's Λ criteria. (3L)

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ST-204: PARAMETRIC INFERENCE

• Introduction: (4 Marks)

- Introduction of Parametric models, Point estimation, Tests of hypotheses and Interval estimation. (1L)
- Joint distribution of a sample and sampling distribution of a Statistic. (2L)
- Likelihood function, examples from standard discrete and continuous models (such as Bernoulli, Poisson, Negative Binomial, Normal, Exponential, Gamma, Pareto etc.) Plotting likelihood functions for these models up to two parameters. (3L)

• Sufficiency: (10 Marks)

- Information in data about the parameters and variation in likelihood function, concept of no information. (1L)
- Sufficiency, Fisher's concept of sufficiency, Sufficient Statistic, Neyman Factorizability criterion, Likelihood equivalence, Minimal sufficient Statistic. (4L)
- Invariance property of sufficiency under one-one transformation of sample space. (1L)
- Exponential families and Pitman families. (3L)
- Fisher information for one and several parameters models. (2L)

• Methods of Estimation: (8 Marks)

- Maximum Likelihood method. (2L)
- Methods of moments and percentiles. (1L)
- Unbiased Estimation. (1L)

• Minimum Variance Unbiased Estimation: (12 Marks)

- UMVUE, Rao-Blackwell Theorem. (2L)
- Completeness property of family of distributions. (2L)
- Lehmann-Scheffe-Rao-Blackwell Theorem and its applications. (2L)
- Necessary and sufficient condition for UMVUE. (1L)
- Cramer-Rao lower bound approach. (2L)

• **Tests of Hypotheses: (20 Marks)**

- Concepts of critical regions. (1L)
- Test functions. (1L)
- Two kinds of errors, Size function, Power function, Level of the test. (2L)
- Introduction of null and alternative hypotheses with examples. (1L)
- Most powerful (MP) and Uniformly Most Powerful (UMP) test in the class of size α tests. (2L)
- Neyman-Pearson Lemma, MP test for simple null against simple alternative hypothesis. (2L)
- UMP tests for simple null hypothesis against one-sided alternatives and for one-sided null against one-sided alternatives in one parameter exponential family. (3L)
- Extensions of these result of Pitman family when only upper or lower end depends on the parameter. (2L)
- MLR property and extension of the above results to the distributions with MLR property. (3L)
- Non-existence of UMP test for simple null against two sided alternatives in one parameter exponential family. (2L)

• **Interval Estimation: (6 Marks)**

- Confidence level, construction of confidence intervals using pivots, Shortest expected length confidence interval. (3L)
- Uniformly most accurate one-sided confidence interval and its relation to UMP test for one-sided null against one-sided alternative hypotheses. (3L)

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ST-205: LINEAR MODELS AND REGRESSION ANALYSIS

• **General Linear Model: (15 Marks)**

- Gauss-Markov set up, Least square estimation, Normal equations, Consistency of system of normal equations and their solution. (3L)
- Estimability of linear parametric function, necessary and sufficient condition for estimability, Best Linear Unbiased Estimator (BLUE). (2L)