॥ अंतरी पेटवू ज्ञानज्योत ॥

NORTH MAHARASHTRA UNIVERSITY, JALGAON.

Syllabus for S.Y.B.Sc.

MATHEMATICS.

(W.E.From June, 2003)

॥ अंतरी पेटवू ज्ञानज्योत ॥

NORTH MAHARASHTRA UNIVERSITY, JALGAON.

CORRECTIONS.

S.Y.B.Sc. Mathematics.

Reference Books for S.Y.B.Sc. Mathematics.

	Paper-I Calculus of Several Variables And Functions of Complex Variables		Paper-II (A) Vector Calculus and Algebra Paper-II (B) Discrete Mathematics and Algebra.
1.	Mathematical Analysis (2e)S.C.Malik, Savita Arora Wiley Eastern Limited.	1.	Vector Analysis M.R.Spiegel Schaum's Outline Series
2.	i) Differential Calculus. ii) Integral Calculus. Both by Kantish Chandra Maity Ramkrishna Ghosh New Central Book Agency(p)Ltd. Kolkata – 700 009.	2.	Vector Analysis N.Ch. S.N.Iyengar. Anmol Publication Pvt. Ltd. New Delhi – 110 002.
3.	i) Calculus ii) Advance Calculus Both by Frank Ayres Schaum's Outline Series.	3.	Topics in Algebra I.N.Herstein Vikas Publication, New Delhi.
4.	Complex Variable M.R.Spiegel Schaum's Outline Series	4.	Modern Algebra Surjeet Singh Qazi Zameeruddin Vikas Publishing House Pvt. Ltd.
5.	Functions of Complex Variable J.N.Sharma Krishna Prakashan Mandir, Meerut.	5.	Modern Algebra J.N.Kapur K.R.Kalra R.Chand & Com.. New Delhi.
		6.	Discrete Mathematical Structures IVth Edition (EEE) B.Kolman, Robert Busby, Ross Prentice Hall of India, New Delhi – 110 001.

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North Maharashtra University
Jalgaon

Syllabus For **S.Y. B.Sc.**

Mathematics

W.E.From June, 2003 .

Paper I : Calculus of several variables and functions of complex variables

Paper II (A) : Vector Calculus and Algebra

OR

Paper II (B) : Discrete Mathematics and Algebra

Paper III : Practical Course based on Paper I & Paper II

North Maharastara University Jalgaon.

Syllabus For S.Y. B.Sc. Mathematics.

With Effect From June,2003.

Paper I

Calculus of Several Variables & Functions of Complex Variables.

1. **Functions of two or three variables:-** **Periods :- 20 Marks :- 21**
   - i) Limits and Continuity.
   - ii) Partial Derivatives. Higher order partial derivatives.
   - iii) Differentiability and Differentials.
   - iv) Necessary and sufficient conditions for Differentiability
   - v) Schwarz's Theorem and Young's Theorem for $f_{xy}(a, b) = f_{yx}(a, b)$
   - vi) Composite functions. Chain Rules.
   - vii) Homogeneous functions. Euler's Theorem.

2. **Mean Value Theorem for a function of two variables :-** **Periods :- 10 Marks :- 11**

   Taylor's Theorem and Maclaurin's Theorem for a function of two variables.

3. **Extreme Values :-** **Periods :- 10 Marks :- 6**

   i) Maxima and Minima.
   ii) Necessary and sufficient conditions for extreme values.

4. **Double and Triple Integrals.** **Periods :- 12 Marks :- 12**

   i) Evaluation of double integral as a repeated integral.
   ii) Area by double integral.
   iii) Evaluation of triple integral as repeated integral.
   iv) Volume by triple integral
   v) Change of order of integration.

5. **Functions of a complex variable.** **Periods:-20 Marks:- 16**

   Limits, Continuity, Derivative, Analytic Function, Cauchy Riemann Equation (Necessary and sufficient Conditions)

6. **Elementary Functions** **Periods:- 8 Marks:- 6**

   Exponential Function of a complex variable. Trigonometric and Hyperbolic Function of a complex variable.
   Logarithmic function of a complex variable.

7. Complex Integration **Periods:-12** **Marks:- 10**

Line Integral, Statement of Cauchy – Gaursat Theorem, Cauchy's Integral Formulea for f(a) & f '(a) .

Cauchy's Integral Formula for $f^{(n)}$ (a) (Without Proof)

Taylor's and Laurent's series (Statement & examples)

8. Calculus of residue. **Periods:-12** **Marks:- 18**

Pole of a function. Residue of a function at its pole. Cauchy's Residue theorem.

Contour Integrations of the type.

(i) $\int_0^{2\pi} f(\cos\theta, \sin\theta)\, d\theta$

(ii) $\int_{-\infty}^{\infty} f(x)\, dx$

(iii) $\int_{-\infty}^{\infty} f(x) \sin mx dx$, $\int_{-\infty}^{\infty} f(x) \cos mx dx$, $m > 0$

where $f(x) = \frac{p(x)}{q(x)}$ is rational and p(x)and q(x) are polynomials in

and p(x)and q(x) are polynomials in x.

Paper II (A)

Vector Calculus and Algebra

1. Vector Function **Periods:- 20** **Marks:- 16**

Vector Function of a single variable. Limits and continuity. Differentiability. Algebra of differentiation. Curves in space. Velocity and acceleration. Vector function of two or three variables. Limits, Continuity. Partial differentiation.

2. Differential operators **Periods:- 20** **Marks:- 16**

Gradient of a scalar point function. Properties of a gradient, Geometrical meaning of a gradient. Directional derivative of a scalar point function. Divergence and Curl of a vector point function and their properties.

3. **Vector Integration** **Periods:-12** **Marks:- 18**

Line integral. Surface integral. Volume integral of a vector point function. Green's theorem in a plane. Stoke's theorem Gauss Divergence theorem and their deductions

4. **Groups:** **Periods:-. 16** **Marks:- 10**

Definition of a group, simple properties of groups abelian groups finite and infinite groups order of a group and power of an element in a group and properties order of an elements in a Group.

5. **Subgroups:** **Periods:- 16** **Marks:- 16**

Definition and criteria for a subset to be subgroup. Cyclic groups. Properties of cosets. Lagranges theorem for finite groups. Euler's theorem and fermat's theorem.

6. **Homomrphisms:** **Periods:-10** **Marks:- 12**

Group homomorphism and its properties. Kernel of a homomorphism and isomorphism of groups.

7. **Rings:** **Periods:-10** **Marks:- 12**

Definition and examples of ring. Simple Properties of ring. commutative ring. Ring with identity element. Boolean ring. Integral domains and fields.

Paper II (B)

Discrete Mathematics And Algebra

1. **Relations and Digraphs** **Period:- 16** **Marks:- 16**

Product set and partitions. Relations and digraphs. Paths in relations and digraphs. Properties of relations. Equivalence relations determined by partition. Operations on relations. Closures: Reflective symmetric and transitive closures. Warshall's Algorithm.

2. **Trees** **Period:- 12** **Marks:- 12**

Trees, Labelled Trees. Tree searching. Undirected trees. Spanning trees of Connected Relations.

3. **Languages and Finite-State Machines. Period:- 12 Marks:- 12**

Languages. Representation of special grammars and languages. Finite state machines & languages. Semi groups, Machines and Languages machines and regular languages.

4. **Groups Code Period:- 12 Marks:- 10**

Coding of binary information and error detection. Decoding and error correction

5. **Groups: Periods:- 16 Marks:- 10**

Definition of a group Simple properties of groups abelian groups finite and infinite groups. Order of a group and power of an element in a group and properties. Order of an elements in a Group.

6. **Subgroups: Periods:- 16 Marks:- 16**

Definition and criteria for a subset to be subgroup. Cyclic groups. Properties of cosets. Lagranges theorem for finite groups. Euler's theorem and Fermat's theorem.

7. **Homomrphisms: Periods:-10 Marks:- 12**

Group homomorphism and its properties. Kernel of a homomorphism and isomorphism of groups.

8. **Rings: Periods:-10 Marks:- 12**

Definition and examples of ring. Simple properties of ring. commutative ring. Ring with identity element. Boolean ring. Integral domains and fields.

Recommended Book For Discrete Mathematics
Discrete Mathematical Structures (IV th Edition)
By Bernard kolman, Robert C Busby, Ross
Prentice Hall & India New Delhi 110001
Eastern Economy Edition (EEE)

Paper III

Practical Course based on Paper I and Paper II

North Maharashtra University Jalgaon

S.Y.B.Sc. Mathematics

Syllabus with limitations and scope

(With effect from June 2003)

Paper I

Calculus of Several Variables & Functions of Complex Variables

1. **Functions of two or three variables.** **Periods :20** **Marks:21**
   - 1.01 Introduction.
   - 1.02 Neighbourhoods.
   - 1.03 Regions.
   - 1.04 Limits and continuity.
   - 1.05 Theorems on limits and continuity (without proofs)
   - 1.06 Partial Derivatives.
   - 1.07 Higher order Partial Derivatives.
   - 1.08 Differentiability and Differentials.
   - 1.09 Necessary and sufficient conditions for differentiability.
   - 1.10 Schwarz's Theorem .
   - 1.11 Young's theorem.
   - 1.12 Composite function. Chain Rules.
   - 1.13 Homogeneous functions.
   - 1.14 Euler's theorem on homogeneous functions.
2. **Mean Value Theorem for a function of two variables.** **Periods :10** **Marks:11**
   - 2.01 Mean Value theorem for a function of two variables.
   - 2.02 Taylor's theorem for a function of two variables.
   - 2.03 Maclaurin's theorem for a function of two variables.
3. **Extreme values.** **Periods :10** **Marks:06**
   - 3.01 Absolute and relative maximum.
   - 3.02 Absolute and relative minimum.
   - 3.03 Relative extremum.
   - 3.04 Theorem (Necessary Condition)
     If f has an extremum at a point (a,b) and if the partial derivatives $f_x(x,y)$ and $f_y(x,y)$ exist in a neighbourhood of (a,b) then $f_x(a,b)=0$ and $f_y(a,b)=0$
   - 3.05 Critical Point.
   - 3.06 Saddle Point.
   - 3.07 Second derivative test for extrema (sufficient conditions) without proof.

4. Double and Triple Integrals. Periods :12 Marks:12

4.01 Introduction.
4.02 Double Integrals
4.03 Evaluation of double integrals (Repeated Integrals)
4.04 Evaluation of Area by double integrals.
4.05 Evaluation of triple integrals (Repeated Integrals)
4.06 Evaluation of volume by triple integrals.
4.07 Change of order of Integration.
  a) Case of constant limits.
  b) Case when limits are definite by an inequality.
  c) Case when limits are functions of variables.

5. Function of complex variable Periods:20 Marks:16

5.01 Introduction
5.02 Neighbourhoods.
5.03 Function of a complex variable.
5.04 Definition of a limit
5.05 Theorem (with proof)

If $f(z) = u(x,y) + i\,v(x,y)$, $z = x + iy$, $z_0 = x_0 + iy_0$, $f(z_0) = u_0 + iv_0$ then

$\lim_{z \to z_0} f(z) = w_0$ iff $\lim_{(x,y) \to (x_0,y_0)} u(x,y) = u_0$ and $\lim_{(x,y) \to (x_0,y_0)} v(x,y) = v_0$

5.06 Theorems on Algebra of limits (without proof)
5.07 Definition of continuity
5.08 Theorems on Algebra of continuity.
5.09 Definition of derivative of f(z)
5.10 Theorem. Every derivable (differentiable) function is continuous but the converse is not true. Give a counter example.
5.11 Definition of Analytic function.
5.12 Necessary and sufficient conditions (with proofs) for function f(z) to be analytic at a point and Cauchy- Riemann's equations.
5.13 Examples of the type that even though the C.R. equations are satisfied, function is not analytic at that point.
5.14 If f(z) is analytic function of z then f(z) is independent of $\bar{z}$.
5.15 Laplace equation, Harmonic function.
5.16 Construction of an analytic function
  f(z) = u + i v I) when u is given
  II) when v is given
  III) by Milne Thomson's method
5.17 Operator $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ and typical examples on it.

6. Elementary Functions. **Periods: 8 Marks:06**

6.01 Exponential function of a complex variable.
6.02 Trigonometric function of a complex variable.
6.03 Hyperbolic function of a complex variable.
6.04 Relations between circular and hyperbolic functions.
$\sin(ix) = i\sinh x$, $\sinh(ix) = i\sin x$
$\cos(ix) = \cos x$, $\cosh(ix) = \cos x$
$\tan(ix) = i\tanh x$, $\tanh(ix) = i\tan x$
and Trignometric identities.
6.05 Logarithmic functions of a complex variable.
6.06 Examples on separation of real and imaginary parts of above functions and related examples.

7. Complex Integration . **Periods:12 Marks: 10**

7.01 Line Integral and theorems on it (without proof)
7.02 Statement and verification of Cauchy-Gaursat's theorem.
7.03 Corollaries on Cauchy-Gaursat's. theorem
7.04 Cauchy's integral formulae for $f(a)$ and $f'(a)$ (with proof)
7.05 Cauchy's General Integral formula for $f^{(n)}(a)$ (without proof)
7.06 Statement of Taylor's and Laurent's series and examples.

8. Calculus of Residues. **Periods:12 Marks: 18**

8.01 Poles of a function
8.02 Residue of a function at its pole
Definition and formulae (without proof) and examples
8.03 Cauchy's Residue theorem (with proof)
8.04 Evaluation of integral by using Cauchy's Residue theorem.
8.05 Contour integrations of the type
(i) $\int_0^{2\pi} f(\cos\theta, \sin\theta)\, d\theta$

(ii) $\int_{-\infty}^{\infty} f(x)\, dx$

(iii) $\int_{-\infty}^{\infty} f(x) \sin mx\, dx$, $\int_{-\infty}^{\infty} f(x) \cos mx\, dx$, $m > 0$

where $f(x) = \frac{p(x)}{q(x)}$ is rational and $p(x)$ and $q(x)$ are polinomials in x.

Paper II (A)

Vector Calculus and Algebra

1. Vector Functions **Period :20** **Marks : 16**

1.01 Introduction
1.02 Definition of a vector point function of a single variable
1.03 Limit of a vector function (Definition and examples)
1.04 Algebra of limits (Statements only)
1.05 Continuity of a vector function (Definition and examples)
1.06 Theorems on continuity (Statements only)
1.07 Derivative of a vector function and differentiability of a vector function
1.08 Theorem : If a vector function $\bar{F} = \bar{F}(t)$ is differentiable at $t=t_0$ then it is continuous at t=to. Show by a counter example that the converse is not true
1.09 Algebra of differentiation of a vector function. Proofs of

i) $\frac{d}{dt}\{\bar{u}+\bar{v}\}$ ii) $\frac{d}{dt}\{\bar{u}\bullet\bar{v}\}$ iii) $\frac{d}{dt}\{\bar{u}\times\bar{v}\}$

iv) $\frac{d}{dt}\{\phi(t)\,\bar{u}\}$ v) $\frac{d}{dt}\{\bar{u}\ \bar{v}\ \bar{w}\}$ vi) $\frac{d}{dt}\{\bar{u}\times(\bar{v}\times\bar{w})\}$

1.10 Chain rule
1.11 Vector differentiation in terms of its components
1.12 If $\bar{u}(t)$ is a constant function on [a, b] iff $\frac{d\bar{u}}{dt} = \bar{O}$
1.13 A differentiable vector point function $\bar{u}(t)$ on [a, b] is of constant magnitude iff $\bar{u}\bullet\frac{d\bar{u}}{dt} = 0 \ \forall t \in [a,b]$
1.14 A non- constant vector function $\bar{u}(t)$ has a constant direction iff $\bar{u}\times\frac{d\bar{u}}{dt} = \bar{0}$
1.15 Curves in space .Unit tangent vector and normal to the curve. Angle between two tangents to the curve . Curvature of a curve at a point
1.16 Velocity and acceleration:
Tangential and normal components of velocity and acceleration
1.17 Vector function of several variables. Vector function of two or three variables

- 1.18 Limits and continuity of vector function of two or three variables.(Definition only)
- 1.19 Partial derivatives (Definition and examples only)
- 1.20 Total differential

2. Differential operates **Periods: 20** **Marks : 16**

- 2.01 Definitions of scalar field, vector field, level surface
- 2.02 Vector differential operator ∇
- 2.03 Gradient of a scalar point function
- 2.04 Properties of gradient

  Let ϕ & Ψ be two scalan point functions

  then i) $\nabla(\phi \pm \Psi) = \nabla\phi \pm \nabla\Psi$

  ii) $\nabla(\phi\Psi) = \phi\nabla\Psi \pm \Psi\nabla\phi$

  iii) $\nabla(C_1\phi \pm C_2\Psi) = C_1 \nabla\phi \pm C_2 \nabla\Psi$ where C_1 & C_2 are constants

  iv) $\nabla\left(\frac{\phi}{\Psi}\right) = \frac{\Psi\nabla\phi - \phi\nabla\Psi}{\Psi^2}$, $\Psi \neq 0$

  v) The scalar point function ϕ is constant iff $\nabla\phi = \bar{0}$
- 2.05 Geometrical meaning of the gradient
- 2.06 Equation of a tangent plane to the surface
- 2.07 Equation of a normal to the surface at a point
- 2.08 Directional derivative of a scalar point function
- 2.09 Divergence of a vector point function
- 2.10 Properties of divergence.

  If $\bar{F}$ & $\bar{G}$ are two vector point functions and ϕ is a scalar point function then

  i) $\nabla\bullet\left(\bar{F} \pm \bar{G}\right) = \nabla\bullet\ \bar{F} \pm \nabla\bullet\ \bar{G}$

  ii) $\nabla\bullet(\phi\ \bar{F}) = (\nabla\phi)\bullet\ \bar{F} + \phi(\nabla\bullet\ \bar{F})$
- 2.11 Physical meaning of divergence of vector point function
- 2.12 Solenoidal vector point function (Definition and example)
- 2.13 Laplacion of a scalon point function (Definition and example)
- 2.14 Curl of a vector point function
- 2.15 Properties curl.

  If $\bar{F}$ & $\bar{G}$ are Two vector point functions and ϕ is a scalar point function then

  i) $\nabla \times \left(\bar{F} \pm \bar{G}\right) = \nabla \times\ \bar{F} \pm \nabla \times\ \bar{G}$

  ii) $\nabla \times (\phi\ \bar{F}) = (\nabla\phi) \times \bar{F} + \phi(\nabla \times\ \bar{F})$
- 2.16 Physical meaning of curl of vector point function
- 2.17 Irrotational vector point function (Definition and example)

2.18 Vector identities.

If $\bar{F}$ & $\bar{G}$ are Two vector point functions and ϕ is a scalar point function then

i) $\nabla \times (\nabla \phi) = \bar{0}$

ii) $\nabla \bullet (\nabla \times \bar{F}) = 0$

iii) $\nabla \times (\nabla \times \bar{F}) = \nabla(\nabla \bullet \bar{F}) - \nabla^2 \bar{F}$

iv) $\nabla \bullet (\bar{F} \times \bar{G}) = \bar{G} \bullet (\nabla \times \bar{F}) - \bar{F} \bullet ((\nabla \times \bar{G})$

v) $\nabla \times (\bar{F} \times \bar{G}) = (\bar{G} \bullet \nabla) \bar{F} - (\bar{F} \bullet \nabla) \bar{G}$

$$+ \bar{F}(\nabla \bullet \bar{G}) - \bar{G}(\nabla \bullet \bar{F})$$

vi) $\nabla(\bar{F} \bullet \bar{G}) = (\bar{G} \bullet \nabla) \bar{F} + (\bar{F} \bullet \nabla) \bar{G}$

$$+ \bar{G} \times (\nabla \times \bar{F}) + \bar{F}(\nabla \times \bar{G})$$

2.19 Conservative field (Definition & examples)

3. **Vector Integration** **Periods :12** **Marks : 18**

3.01 Line Integral (Definition & examples)
3.02 Surface Integral (Definition & examples)
3.03 Volume Integral (Definition & examples)
3.04 Green theorem in a plane for simple closed curve (with proof)
3.05 Stoke's theorem (without proof)
3.06 Gauss Divergence theorem (without proof)
3.07 Some dedications on Stroke's and Gauss Divergence theorem and examples.

4. **Groups.** **Periods : 16** **Marks : 10**

4.01 Definition of a group and examples
4.02 Properties of groups.

Theorem 1. If G is a group then
i) Identity of G is unique
ii) Every element of G has unique inverse
iii) $(a^{-1})^{-1} = a, \forall a \in G$
iv) $(ab)^{-1} = b^{-1} a^{-1}, \forall a, b \in G$

Theorem 2. Cancellation laws in a group G.

Theorem 3. If G is a group and $a, b \in G$ then the equations
(i) $ax = b$ (ii) $ya = b$ have unique solutions in G.

4.03 Abelian groups, finite and infinite groups and examples.

4.04 Integral power of an element in a group.

Theorem1. Let G be a group and a, b $\in$ G.

then (i) $a^m a^n = a^{m+n}$, (ii) $(a^m)^n = a^{mn}$, $\forall$ m, n $\in$ N, Z.

Theorem 2. Let G be a group and a, b $\in$ G be such that ab=ba.

then $(ab)^n = a^n b^n$, $\forall$ $n \in Z$.

4.05 Order of an element in a group.

Theorem 1.The order of every element in a finite group is finite.

Theorem 2.Let G be a group and a,b $\in$ G. Then

(i) $O(a^{-1}) = O(a)$

(ii) $O(a) = O(b^{-1}ab)$

(iii) $O(ab) = O(ba)$

5. Subgroups **Periods : 16** **Marks : 16**

5.01 Definition of a subgroups and examples.

Theorem 1. A non - empty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H$, $\forall$ a,b $\in$ H

Theorem 2. A non - empty subset H of a group G is subgroup of G if and only if (i) a, b $\in$ H $\Rightarrow$ ab $\in$ H and ii) a $\in$ H $\Rightarrow$ $a^{-1} \in H$

Theorem 3. Let (G, $*$) be a finite group. show that a non-empty subset H of G is subgroup of G if and only if a ,b $\in$ H $\Rightarrow$ $a * b \in H$

Theorem 4. (i) Intersection of two subgroups of a group is a subgroup

(ii) Let G be a group and H, K be subgroups of G Prove that $H \cup K$ is a subgroup of G if and only if $H \subseteq K$ or $K \subseteq H$

5.02 Definition of cyclic group and cyclic subgroup and examples.

Theorem 1.Every cyclic group is abelian

Theorem 2. Every subgroup of a cyclic group is cyclic.

5.03 Definition of right coset and left coset

Theorem 1. Let G be a group and H a subgroup of G. Prove that

(i) He = H = eH (ii) (Ha) b = H (ab),

and b (aH) = (ba) H, $\forall$ a, b $\in$ G.

(iii) If G is abelian then Ha = aH $\forall$ a $\in$ G

Theorem 2. Let H be a subgroup of a group G. Prove that

(i) a $\in$ H $\Leftrightarrow$ Ha = H = aH (ii) Ha = Hb $\Leftrightarrow$ ab^{-1} a $\in$ H and aH = bH $\Leftrightarrow$ $b^{-1}a \in H$, $\forall$ a, b $\in$ G.

(iii) Any two right (left) coset of a subgroup H of group G are either disjoint or identical.

5.04 Theorem 1. Lagrange's theorem for finite groups.

Theorem 2. Prove that every group of prime order is cyclic and hence abelian.

Theorem 3. Order of every element of a finite group is a divisor of order of a group.

Theorem 4. If G is a finite group and $a \in G$ then $a^{o(G)} = e$.

Theorem 5. Euler's theorem

Theorem 6. Fermat's theorem

Periods : 10 Marks : 12

6. Homomorphism:

6.01 Definition of a group homomorphism and examples

Theorem 1. Let $f : G \rightarrow G'$ be a group homomorphism.

Prove that (i) If e is identity of G then f(e) is identity of G'

(ii) $f(a^{-1}) = (f(a))^{-1}, \forall a \in G$

(iii) $f(a^m) = (f(a))^m, \forall m \in Z$, where $a \in G$.

Theorem 2. Let $f: G \rightarrow G'$ be a group homomorphism.

Prove that (i) If H is a subgroup of G then f (H) is a subgroup of G'

(ii) If H' is a subgroup of G' then $f^{-1}(H')$ is a subgroup of G

Theorem 3. Prove that the homomorphic image of an abelian group is abelian.

Theorem 4. Prove that the homomorphic image of cyclic group is cyclic.

6.02 Kernel of a homomorphism and examples

Theorem: Let $f : G \rightarrow G'$ be a group homomorphism

Prove that (i) Ker f is a subgroup of G

(ii) f is one-one if and only if Ker f = {e}, where e is the identity element of G.

6.03 Definition of isomorphism of groups and examples.

Theorem 1. The relation of isomorphism of groups is an equivalence relation

Theorem 2. Let $f : G \rightarrow G'$ be a isomorphism and $a \in G$. show that $O(a) = O(f(a))$

Theorem 3. (i) Every finite cyclic group of order n is isomorphic to $(Z_n, +_n)$.

(ii) Every infinite cyclic group is isomorphic to(Z,+)

Periods : 10 Marks : 12

7. Rings.

7.01 Definition and examples of ring, Commutative ring, Ring with identity element

Theorem 1. If R is a ring then for all $a, b \in R$,

i) $a0 = 0 = 0a$

ii) $a(-b) = (-a)b = -(ab)$

iii) $(-a)(-b) = ab$

Theorem 2. If R is a ring with identity 1 then

i) $(-1)\,a = -a,\ \forall a \in R$

ii) $(-1)(-1) = 1$.

7.02 Definition of a Boolean ring.

Theorem. Prove that every Boolean ring is a commutative ring.

7.03 Definition and examples of integral domain and field.

Theorem 1. Prove that a commutative ring R is an integral domain if and only if $ab = ac \Rightarrow b = c$, where $a, b, c \in R$, $a = o$.

Theorem 2. Prove that a commutative ring R is an integral domain if and only if $ab = o \Rightarrow a = o$ or $b = o$, where $a\,b \in R$.

Theorem 3. Prove that every field is an integral domain.

Theorem 4. Prove that every finite integral domain is a field.

PAPER II (B)

Discrete Mathematics and Algebra

1. Relations and Digraphs **Period : 16** **Marks : 16**

1.01 Product Set

1.02 Partition (Quotient Set)

1.03 Relation from A to B. Subset of A x B. Domain and Range of relation

1.04 Relative Sets R(a), a in A and R(B), B is subset of A

1.05 Matrix of a relation. Boolean matrix and Boolean product.

1.06 Diagraph of a relation ; pictorial representation of a relation.

1.07 Path of length n from a to b in a relation R

1.08 $xR^n y$ (R is relation on A)

1.09 $xR^\infty y$ (Connectivity relation for R)

1.10 Theorem 1

If R is a relation on $A=\{a_1, \ldots\ldots\ldots\ldots, a_n\}$

then $M_R{}^2 = M_R \odot M_R$ where $\odot$ is the Boolean product.

Theorem 2

For $n \geq 2$ and R is a relation on a finite set A then

$M_R n = M_R \odot M_R \odot \ldots\ldots\ldots\ldots \odot\ M_R$ (n factors)

1.11 Reachability relation and composition of paths in a relation

1.12 Reflexive and Irreflexive, symmetric, asymmetric and antisymmetric and transitive relation. Definition and examples related with matrix relation.

1.13 Equivalence relation determined by a partition. Method of finding partition A/R where A is finite and countable and R is relation on A.

1.14 Operations on relations ; complementary relation , inverse relation.
(Definition and examples)

1.15 Closures . Reflexive , Symmetric and transitive closures .

1.16 Theorem 1 . Let R be a relation on a set A then R^{∞} is a closure of A
Theorem 2. Let A be a set with | A | = n and R is a relation on A then $R^{\infty} = R \cup R^2 \cup \ldots \cup R^n$

1.17 Warshall's Algorithm and
Theorem . If R and S are equivalence relations on a set A then the smallest relation containing both R and S is $(R \cup S)^{\infty}$

2. Trees. **Periods : 12 Marks : 12**

2.01 Tree , Root of a tree. Rooted tree (T, V_0) where T is tree with root V_0 , T being a relation on A & V_0 is in A.

2.02 Theorem : Let (T,V_0) be a rooted tree then,
a) there are no cycles in T
b) V_0 is the only root of T
c) Each Vertex in T , other than V_0 has in-degree one, and V_0 has in - degree zero .

2.03 Level , Height of a tree , parent of spring , leaves, ordered tree.

2.04 n-tree , complete n-tree, Binary tree , complete binary tree.
Theorem : If (T,V_0) is a rooted tree and $V \in T$ then T(v) is also rooted tree with root v where T(v) is sub tree of T beginning at v.

2.05 Lebelled trees, positional binary tree

2.06 Tree searching . Algorithm pre order, Algorithm in order, Algorithm post order searching general trees.

2.07 Undirected trees. Simple path, Simple cycle, Connected symmetric relation, acyclic relation.

2.08 Spanning trees of connected relations
1) Prim's Algorithm for finding a spanning tree for symmetric connected relation R on infinite set A
2.) Algorithm for finding the matrix of relation R' where R' is obtained by merging vertices a & b in new vertex a'

3. Languages and Finite State Machines. Periods : 12 Marks : 12

3.01 Languages. Strings, Word. Sentence, Syntax , Sematics.

3.02 Grammer phase structure grammer production. Direct derivaribility. Terminal symbols and non-terminal symbols.

3.03 Derivation of a sentence. Language of a grammer , Derivation tree for a sentence.

3.04 Types 0,1,2,3. Pharase structure grammers, context free grammer. Regular grammer. Parsing.

3.05 BNF notation and syntax diagrams

3.06 Regular Grammers and Regular expressing

Theorem. Let S be a finite set and $L \subseteq S^*$ Then L is a regular set iff $L = L(G)$ for some regular grammer $G = (V, S, V_o, \to)$ (without proof)

3.07 Finite state machines. State. State translation function. State translation table . Relation R_M on S.

3.08 Moor Machine, Moor congruence R on M and Quotient of M w.r.t. R

3.09 State translation function f_w where $w = x_1x_2 \ldots x_n$,

$fw = f_{x_n} o f_{x_{n-1}}, \ldots\ldots\ldots f_{x_1}, \qquad f_n = 1_s$

Theorem : Let $M = (S, I, F,)$ be a finite state machine. Define $T : I^* \to S^*$ by $T(W) = f_w$, $W \neq \Lambda$ and $T(\Lambda) = 1_s$ then

a) w_1 and w_2 are in I^* then $T(w_1 . w_2) = T(w_2) . T(w_1)$

b) $M = T (I^*)$ then M is a sub monoid of S^S

Monoid of a machine. Language of machine M

3.10 W compatible. Equivalent machines Procedure for reducing a given Moore machine to equivalent machine.

4. Groups Code **Periods : 12** **Marks : 10**

4.01 Message, Word

4.02 Encoding Function, code word, weight of x

4.03 Parity check code

4.04 Hamming distance between x and y

4.05 Properties of distance function

Theorem : If x,y,z be the elements of B then

i) $\delta (x,y) = \delta (y,x)$

ii) $\delta (x,y) \geq 0$

iii) $\delta (x,y) = 0$ iff $x = y$

iv) $\delta (x,y) \leq \delta (x,z) + \delta (z,y)$

4.06 Minimum distance of (m,n) encoding function

Theorem : A_n (m,n) encoding function e: $B^m \to B^n$ can detect k for fewer errors iff its minimum distance is atleast k + 1

4.07 Group code

Let e: $B^m \to B^n$ be a group code. The mi[illegible] distance of e is the minimum weight of a non-ze[illegible]

4.08 mod 2 - sum $D \oplus E$ as $D \oplus E$

& mod 2 - Boolean product of [illegible]

Distributive property for ⊕ & * (statement)
Theorem :- Let m & n be non-negative integers with $m < n$, $r = n-m$ & H be $n \times r$ Boolean matrix . Then the function $f_H : B^n \rightarrow B^r$ defined by
$f_H(x) = x * H$, $x \in B^n$ is a homomorphism from group B^n to group B^r
Theorem Let $x = y_1 y_2 \ldots y_m x_1 \ldots x_r \in B^n$
then $x * H = \bar{0}$ iff $x = e_H(b)$ for some $b \in B^m$

4.09 Group code e_H corresponding to parity check matrix H
4.10 (n, m) decoding function.
4.11 Maximum liklihood decoding function d associated with e
Theorem : Let e be (m , n) encoding function and d is the maximum liklihood decoding function associated with e . The (e, d) can correct k or fewer errors iff the minimum distance e is atleast 2k+1
4.12 Decoding procedure for a group code.
4.13 Decoding procedure for a group code given by a parity matrix.

5. Groups. Periods : 16 Marks : 10

5.01 Definition of a group and examples
5.02 Properties of groups.
Theorem 1. If G is a group then
i) Identity of G is unique
ii) Every element of G has unique inverse
iii) $(a^{-1})^{-1} = a$, $\forall a \in G$
iv) $(ab)^{-1} = b^{-1} a^{-1}$, $\forall a, b \in G$
Theorem 2. Cancellation laws in a group G.
Theorem 3. If G is a group and $a, b \in G$ then the equations
(i) $a x = b$ (ii) $y a = b$ have unique solutions in G.
5.03 Abelian groups, finite and infinite groups and examples.
5.04 Integral power of an element in a group.
Theorem1. Let G be a group and $a, b \in G$.
then (i) $a^m a^n = a^{m+n}$ (ii) $(a^m)^n = a^{mn}$, $\forall m, n \in N, Z$.
Theorem 2. Let G be a group and $a, b \in G$ be such that $ab=ba$.
then $(ab)^n = a^n b^n$, $\forall n \in Z$.
5.05 Order of an element in a group.
Theorem 1.The order of every element in a finite group is finite.
Theorem 2.Let G be a group and $a,b \in G$. then
(i) $O(a^{-1}) = O(a)$
(ii) $O(a) = O(b^{-1}ab)$
(iii) $O(ab) = O(ba)$

6. **Subgroups** **Periods : 16** **Marks : 16**

6.01 Definition of a subgroups and examples.

Theorem 1. A non - empty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H \; \forall \; a,b \in H$

Theorem 2. A non - empty subset H of a group G is subgroup of G if and only if (i) $a, b \in H \Rightarrow ab \in H$ and ii) $a \in H \Rightarrow a^{-1} \in H$

Theorem 3. Let (G, $*$) be a finite group. Show that a non-empty subset H of G is subgroup of G if and only if $a, b \in H \Rightarrow a * b \in H$

Theorem 4. (i) Intersection of two subgroups of a group is a subgroup

(ii) Let G be a group and H, K be subgroups of G Prove that $H \cup K$ is a subgroup of G if and only if $H \subseteq K$ or $K \subseteq H$

6.02 Definition of cyclic group and cyclic subgroup and examples.

Theorem 1. Every cyclic group is abelian

Theorem 2. Every subgroup of a cyclic group is cyclic.

6.03 Definition of right coset and left coset

Theorem 1. Let G be a group and H a subgroup of G. Prove that

(i) $He = H = eH$ (ii) $(Ha)b = H(ab)$, and $b(aH) = (ba)H, \forall a, b \in G$.

(iii) If G is abelian then $Ha = aH, \forall a \in G$

Theorem 2. Let H be a subgroup of a group G. Prove that

(i) $a \in H \Leftrightarrow Ha = H = aH$ (ii) $Ha = Hb \Leftrightarrow ab^{-1} \in H$, and $aH = bH \Leftrightarrow b^{-1}a \in H, \forall a, b \in G$.

(iii) Any two right (left) coset of a subgroup H of group G are either disjoint or identical.

6.04 Theorem 1. Lagrange's theorem for finite groups.

Theorem 2. Prove that every group of prime order is cyclic and hence abelian.

Theorem 3. Order of every element of a finite group is a divisor of order of a group.

Theorem 4. If G is a finite group and $a \in G$ then $a^{o(G)} = e$.

Theorem 5. Euler's theorem

Theorem 6. Fermat's theorem

7. **Homomorphism:** **Periods : 10** **Marks : 12**

7.01 Definition of a group homomorphism and examples

Theorem 1. Let $f : G \to G'$ be a group homomorphism.

Prove that (i) If e is identity of G then f (e) is identity of G'

(ii) $f(a^{-1}) = (f(a))^{-1}$, $\forall a \in G$

(iii) $f(a^m) = (f(a))^m$, $\forall m \in Z$, where $a \in G$.

Theorem 2. Let f: G → G' be a group homomorphism.

Prove that (i) If H is a subgroup of G then f (H) is a subgroup of G'

(ii) If H' is a subgroup of G' then $f^{-1}(H')$ is a subgroup of G

Theorem 3. Prove that the homomorphic image of an abelian group is abelian.

Theorem 4. Prove that the homomorphic image of cyclic group is cyclic.

7.02 Kernel of a homomorphism and examples

Theorem: Let f : G → G' be a group homomorphism

Prove that (i) Ker f is a subgroup of G

(ii) f is one-one if and only if Ker f = {e}, where e is the identity element of G.

7.03 Definition of isomorphism of groups and examples.

Theorem 1. The relation of isomorphism of groups is an equivalence relation.

Theorem 2. Let f : G → G' be a isomorphism and a ∈ G. show that O (a) = O (f (a))

Theorem 3. (i) Every finite cyclic group of order n is isomorphic to $(Z_n, +_n)$.

(ii) Every infinite cyclic group is isomorphic to (Z, +)

8. Rings. Periods : 10 Marks : 12

8.01 Definition and examples of ring, Commutative ring, Ring with identity element.

Theorem 1 If R is a ring then for all a, b ∈ R,

i) a.o = o = o.a

ii) a (-b) = (-a)b = - (ab)

iii) (-a) (-b) = ab.

Theorem 2. If R is a ring with identity 1 then

i) (-1) a = -a, ∀a ∈ R

ii) (-1) (-1) = 1.

8.02 Definition of a Boolean ring.

Theorem. Prove that every Boolean ring is a commutative ring.

8.03 Definition and examples of integral domain and field.

Theorem 1. Prove that a commutative ring R is an integral domain if and only if ab = ac ⇒ b = c , where a , b ,c ∈ R, a ≠ o.

Theorem 2. Prove that a commutative ring R is an integral domain if and only if ab = o ⇒ a = o or b=o, where a b ∈ R.

Theorem 3. Prove that every field is an integral domain.

Theorem 4. Prove that every finite integral domain is a field.

Paper III

Practical Course Based on Paper I & Paper II

INDEX

Practical No. -1:
Limits, Continuity, Partial Derivatives and Diffentiability.

1. a) Evaluate (i) $\lim_{(x,y)\to(0,0)} \dfrac{xy^4}{x^2+y^4}$

(ii) $\lim_{(x,y)\to(0,0)} \dfrac{x\sin x + y\sin y}{x^2+y^2}$

b) Show that the repeated limits $\lim_{x\to 0}\lim_{y\to 0} f(x,y)$ and $\lim_{y\to 0}\lim_{x\to 0} f(x,y)$ exist,

where $f(x,y) = \dfrac{y-x}{y+x} \cdot \dfrac{1+x}{1+y}$

2. i) Show that $f(x,y) = \dfrac{x^4y^4}{(x^2+y^4)^3}$ when $(x,y) \neq (0,0)$ and $f(x,y) = 0$ is continuous at (0, 0).

ii) Discusss the continuity of $f(x,y)$ at (2,1) if

$$f(x,y) = \frac{\sin^{-1}(xy-2)}{\tan^{-1}(3xy-6)}, \quad xy \neq 2$$
$$= \frac{1}{3}, \quad xy = 2$$

3. i) If $x^x y^y z^z = c$, show that $\dfrac{\partial^2 z}{\partial x\,\partial y} = -(x\log_e x)^{-1}$

ii) If $u = (x^2+y2+z^2)^{-1/2}$, $x^2+y^2+z^2 \neq 0$.

show that $\dfrac{\partial u}{\partial x}^2 + \dfrac{\partial u}{\partial y}^2 + \dfrac{\partial u}{\partial z}^2 = u^4$

4. i) Let $f(x,y) = (x^2+y^2)\tan^{-1}\dfrac{y}{x}$, when $(x,y) \neq (0,0)$

$= \dfrac{\pi y^2}{2}$, when x= 0 but y may not be 0.

show that $f_{xy}(x,y) = f_{yx}(x,y) = \dfrac{x^2-y^2}{x^2+y^2}$ where $(x,y) \neq (0,0)$

but $f_{xy}(0,0) \neq f_{yx}(0,0)$.

ii) Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$, where

$f(x,y) = \dfrac{x^3y}{x^2+y^2}$, when $x^2+y^2 \neq 0$ and $f(0,0) = 0$.

5\. i) Show that the function $f(x,y) = \sqrt{(xy)}$ has first partial derivatives at the origin but is not differntiable there.

ii) If

$$f(x,y) = \frac{2x^3-3y^3}{x^2+y^2}, \text{ when } x \neq 0, y \neq 0$$

$f(x,y) = 0.$

a) Evaluate $f_x^{(0,0)}$ and $f_y^{(0,0)}$.

b) Is the function continuous at (o,o) ?

c) Is the function differentiable at (o,o) ?

Practical No. 2 .

Differentials Composite Function and Homogeneous Functions

1\. i] Use differentials to find the approximate value of $\sqrt{125\,(17)}^{\,1/4}$

ii] By using differentials find the approximate value of f (5.12 , 6.85) where $f(x,y) = x^2y - 3y$

2\. i] If $u = F(x^2-y^2, y^2-z^2, z^2-x^2)$, Prove that

$$\frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial u}{\partial y} + \frac{1}{z}\frac{\partial u}{\partial z} = 0$$

ii] if $z=f(x,y)$, where $x=e^u \cos v$, $y = e^u \sin v$,
Show that

$$y\frac{\partial z}{\partial u} + x\frac{\partial z}{\partial v} = e^{2u}\frac{\partial z}{\partial y}.$$

3\. i] $z = f(x, y)$ and $x=r\cos\theta$, $y = r\sin\theta$, Prove that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left[\frac{\partial z}{\partial \theta}\right]^2$$

ii] $x= r\cos\theta$,$y = r\sin\theta$, Show that

$$\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} = \frac{1}{r}\left[\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2\right]$$

4. Verify Euler's Theorem for the function
(i) $z = x^n \sin \dfrac{y}{x}$. ii) $f(x,y) = (x+y)\log\left(\dfrac{y}{x}\right)$

5. i] if $e^u = x^3 + y^3 - x^2y \quad xy^2$. find the values of

a) $x \dfrac{\partial u}{\partial x} + y \dfrac{\partial u}{\partial y}$

b) $x^2 \dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2}$

ii] $u = \operatorname{cosec}^{-1}\left(\dfrac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}} \right)$

Find the value of

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$$

Practical No.3
Mean Value Theorems and Extreme Values

1. a) Expand $e^x \log(1+y)$ in powers of x and y upto terms of third degree.

   b) Expand $f(x,y) = \tan^{-1}(y/x)$ in powers of $(x-1)$ and $(y-1)$ upto third degree terms. Hence compute $f(1.1, 0.9)$ approximately.

2. a) Express $x^3 + 3xy^2 + 5y^3$ in powers of $(x-1)$ and $(y+2)$

   b) If $f(x,y) = x^2y + 2xy^2$ show that θ used in M.V.T applied to the point (1,2) and (3,3) satisfies the quadratic equation
   $12\theta^2 + 30\theta - 19 = 0$

3. a) Expand $f(x,y) = \sin xy$ in powers of $(x-1)$ and $(y-\pi/2)$ upto and including second degree terms.

   b) Find the maximum and minimum values of
   $f(x,y) = 2(x^2 - y^2) - x^4 + y^4$

4. a) Find the minimum distance of the origin from the plane
   $3x + 2y + z = 12$

   b) Investigate the maximum and minimum values of the function
   $f(x,y) = x^3 + y^3 - 3x - 12y + 20$

5. a) A rectangular box open at the top is to have volume 32 cubic feet. Find the dimensions of the box requiring least material for its construction.

   b) Find the points on the surface $z^2 = xy + 1$ nearest to the origin.

Practical No. 4
Multiple Integrals.

1. a) Evaluate $\iint 1/(x^4 + y^2)\, dx\, dy$ over the region $y \geq x^2, x \geq 1$.

   b) Evaluate $\iint y\, dx\, dy$ over the area bounded by $y = x^2$ and $x + y = 2$

2. a) Evaluate $\int_0^1 \int_0^{\sqrt{(1-x^2)}} dx\, dy \;/\; (1 + e^y)\, \sqrt{(1-x^2-y^2)}$

   b) Change the order of integration in the integral.

   $$\int_{-a}^{a} \int_0^{\sqrt{(a^2-y^2)}} f(x,y)\, dx\, dy$$

3. a) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ using double integral.

   b) Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$

4. a) Evaluate $\iiint \frac{dx\, dy\, dz}{(1 + x + y + z)^3}$ over the volume of tetrahedron bounded by $x = 0, y = 0, z = 0$ and $x + y + z = 1$

   b) Evaluate $\int_0^1 \int_0^{\sqrt{(1-x^2)}} \int_0^{\sqrt{(1-x^2-y^2)}} xyz\, dx\, dy\, dz$

5. a) Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ by using triple integral.

   b) Using triple integral find the volume of the ellipsoid

   $$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Practical No. 5 :

Revision

Practical N0. 6

Limit, Continuity & Analytic Functions

1 Evaluate i) $\lim\limits_{z \to -i} \dfrac{8z^5 \quad 3z^4 + 16z^3 + 8z + 3}{z + i}$

ii) $\lim\limits_{z \to 1+i} \dfrac{(z^2 + 4)(1 + i - z)}{z^2 - 2iz + 2i - 2z}$

2. a) $f(z) = \dfrac{z^2 + z + 1}{z - e^{2\pi i/3}} \quad , z \neq {}^{2\pi i}/_{3}$

$= 1 + i\sqrt{3} \quad ; z = {}^{2\pi i}/_{3}$

Discuss continuity of $f(z)$ at $z = {}^{2\pi i}/_{3}$

b) Discuss continuity at $z = o$

$f(z) = \dfrac{x^4 y\,(iy - x)}{x^8 y^2 \; \bar{z}} \quad , z \neq o$

$= 0 \quad , z = o$

3. Let $f(z) = \dfrac{x^3(1+i) - y^3(1-i)}{x^2 + y^2} \quad ; z \neq o$

$= 0 \quad ; z \neq o$

Show that $f(z)$ is continuous at origin, also C.R. equations are satisfied at the origin but $f(z)$ is not analylic at the origin.

4. If $f(z) = u + iv$ is analytic function of z where $u - v = e^x(\cos y - \sin y)$. find $f(z)$ in terms of z

5. Show that $\nabla^2 |f(z)|^2 = 4|f'(z)|^2$

where ∇^2 is Laplacian operator and $f(z) = u + iv$ is analytic function.

Practical - 7
Elementary functions Taylor's & Laurent's series

1. $u = \log\{\tan(\pi/4 + x/2)\}$ prove that $\tan h\, u/2 = \tan x/2$ and $\cos h\, u = \sec x$
2. If $\sin^{-1}(\alpha + i\beta) = u + iv$, prove that $\sin^2 u$ and $\cosh^2 v$ are the roots of the quadratic equation $\lambda^2 - (1+\alpha^2 + \beta^2)\lambda + \alpha^2 = 0$
3. show that

   a) $\sin h\, 2z = 2 \sin h\, z \cos h\, z$

   b) $\cos h\, 2z = \cosh^2 z + \sin h^2 z$

   c) $\tan[i \log (a-ib)/(a+ib)] = 2ab / a^2 - b^2$
4. If $x + iy = \cos h(u + iv)$, show that $x^2/\cos h^2 u + y^2/\sin h^2 u = 1$ and $x^2/\cos^2 v - y^2/\sin^2 v = 1$
5. Obtain the expansion of $f(z) = (z^2 - 1)/(z+2)(z+3)$ in powers of z in the region a) $|z| < 2$ b) $2 < |z| < 3$ c) $|z| > 3$

Practical – 8
Line Integral & Cauchy's integral formulae.

1. Evaluate $\int_0^{2+i} z^2\, dz$

   a) along a straight line joining o to 2+i

   b) first along the straight line joining o to 2 and then along the straight line joining 2 to 2 + i
2. a) Evaluate $\int_C \bar{z}\, dz$, where C is the curve from $z = 0$ to $z = 4 + 2i$ given by $z = t^2 + it$ where t is a parameter.

   b) Evaluate $\int_C (x+2y)dx + (y - 2x)\, dy$ where C is an ellipse $x^2/16 + y^2/9 = 1$ described in counter clock wise sense.
3. Evaluate $\int_C (3z+1)/(z^2 - 2z - 3)\, dz$ where C is $|z| = 4$ by using Cauchy's integral formula.
4. Using Cauchy's integral formula, evaluate $\int_C \frac{\sin^6 z}{(z-\pi/6)^3} dz$ where C is $|z| = 1$
5. Evaluate $\int_C \frac{e^z}{z^2 + 1} dz$

where a) C is the circle $|z - i| = 1$ b) C is the circle $|z| = ½$

Practical – 9
Calculus of Residues

1. Find the poles and residues at the poles for the functions
   a) $(3z^2+2)/(z-1)(z^2+9)$ b) $[(z+1)/(z-1)]^2$
2. Using Cauchy's residue theorem evaluate $\int_C dz/z^3(z+4)$
   where C is $|z| = 2$
3. Show, by using, by contour integration, that
   $$\int_0^{2\pi} \{\cos 2\theta/(5+4\cos\theta)\}d\theta = \pi/6$$
4. Evaluate, by contour integration,
   $$\int_0^{\infty} \{x^2/(x^2+1)(x^2+4)\}\,dx$$
5. Show that
   $$\int_0^{\infty} \{\cos mx/(x^2+1)\}\,dx = (\pi/2)\,e^{-m},\ m > 0.$$

Practical – 10
Revision

Practical – 11(A)
Differentiation of Vector Functions.

1. **i) If** $\bar{r} = a\cos t\ \bar{i} + a\sin t\ \bar{j} + a\,t\tan\alpha\ \bar{k}$
   find a) $\frac{d\bar{r}}{dt} \times \frac{d^2\bar{r}}{dt^2}$ b) $\left[\frac{d\bar{r}}{dt}\ \frac{d^2\bar{r}}{dt^2}\ \frac{d^3\bar{r}}{dt^3}\right]$

   ii) If $\bar{f}(t) = 2\bar{i} - \bar{j} + 2\bar{k}$ when $t = 2$ and
   $\bar{f}(t) = 4\bar{i} - 2\bar{j} + 3\bar{k}$ when $t = 3$
   show that $\int_2^3 \bar{f}\cdot\frac{d\bar{f}}{dt} = 10$

   iii) Show that $\bar{r} = \bar{a}e^{kt} + \bar{b}e^{lt}$ is solution of the differential equation $\frac{d^2\bar{r}}{dt^2} + p\frac{d\bar{r}}{dt} + q\bar{r} = 0$
   where p, q are constants, k, l are the distinct roots of $m^2 + pm + q = 0$, $\bar{a}$, $\bar{b}$ are arbitrary constant vectors.

2. i) If $\bar{r} = x\cos y\ \bar{i} + x\sin y\ \bar{j} + c\log[x + \sqrt{(x^2-c^2)}]\ \bar{k}$,

find the unit vector perpendicular to both $\frac{\partial \bar{r}}{\partial x}$ and $\frac{\partial \bar{r}}{\partial y}$

ii) If $\bar{r} = \frac{a}{2}(x+y)\,\bar{i} + \frac{b}{2}(x-y)\,\bar{j} + xy\,\bar{k}$

a, b are constants.

Find $\left[\frac{\partial \bar{r}}{\partial x} \; \frac{\partial \bar{r}}{\partial y} \; \frac{\partial^2 \bar{r}}{\partial x^2} \right], \left[\frac{\partial \bar{r}}{\partial x} \; \frac{\partial \bar{r}}{\partial y} \; \frac{\partial^2 \bar{r}}{\partial x \partial y} \right]$

3. i) Find the unit tangent vector and curvature at a point P(x, y, z) on the curve $x = a\cos\theta$, $y = a\sin\theta$, $z = a\theta\tan\alpha$ where a and α are constants.

ii) A particle moves along the curve $\bar{r} = 2t^2\,\bar{i} + (t^2 - 4t)\,\bar{j} + (3t-5)\,\bar{k}$, obtain the components of its velocity and acceleration at t = 1 along the direction $\bar{i} - 3\bar{j} - 2\bar{k}$

iii) A Particle moves along the curve $\bar{r} = e^{t}\,\bar{i} + e^{-t}\,\bar{j} + t\sqrt{2}\,\bar{k}$. find the magnitude of the tangential and normal components of its acceleration.

iv) A particle moves along a plane curve $x = \sec t$, $y = \tan t$. Find its velocity and acceleration at $t = \pi/6$

Practical No. 12(A)
Differential operators.
Gradient of scalar functions, Divergence and curl of vector functions

1. i) Find the equation of tangent plane and normal to the surface $z = x^2 + y^2$ at the point (2,-1,5)

ii) Find the acute angle between the surface $xy^2z = 3x + z^2$ and $3x^2 - y + 2z = 1$ at the point (1,-2,1)

iii) Find the constants a and b so that the surfaces $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1,-1,2)

2. i) Find the directional derivative of $\phi(x,y,z) = x^2y + xz^2 - 2$ at

A(1,1,-1) along AB where B = (2,-1,3)

ii) Find the values of a,b,c so that the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at (1,2,-1) has a maximum magnitude 64 in the direction parallel to z-axis

3. i) Find div F and curl F at (4,3,2) where $\bar{F} = x^2y\,\bar{i} + xz\,\bar{j} + yz\,\bar{k}$

ii) Show that $\nabla^2 f(r) = (2/r)\, f'(r) + f''(r)$ and determine f(r) such that $\nabla^2 f(r) = 0$, where $\bar{r} = x\,\bar{i} + y\,\bar{j} + z\,\bar{k}$

4. i) If $\bar{r} = x\,\bar{i} + y\,\bar{j} + z\,\bar{k}$ and $r = |\bar{r}|$
Find a) $\text{div}\,(r^n\,\bar{r})$ b) $\text{curl}\,(r^n\,\bar{r})$ c) $\text{curl}\,[(\bar{a} \times \bar{r})\,r^n]$

ii) Prove that $\nabla \cdot \left[\frac{f(r)}{r}\,\bar{r}\right] = \frac{1}{r^2}\frac{d}{dr}\left[r^2 f(r)\right]$

Practical No. 13(A)
Vector Integration.

1. i) Show that the function $\bar{f} = (2xy + z^3)\,\bar{i} + x^2\,\bar{j} + 3xz^2\,\bar{k}$ is a conservative field. Find the scalar potential ϕ such that $\nabla\phi = \bar{F}$

ii) Calculate the work done when a force $\bar{f} = (2x-y+4)\,\bar{i} + (5y + 3x - 6)\,\bar{j}$ moves a particle in the xy-plane around a triangle C with vertices at (0,0),(3,0),(3,2) traversed in the counter clock - wise sense.

2. i) Evaluate $\iint_S \bar{f} \cdot \bar{n}\, ds$ where $\bar{f} = 18z\,\bar{i} - 12\,\bar{j} + 3y\,\bar{k}$ and S is the part

of the plane 2x +3y+6z=12 in the first octant

ii) find the flux of $\bar{f} = y\,\bar{i} + 2x\,\bar{j} - z\,\bar{k}$ over the surface S of the plane 2x +y=6 in the first octant cut off by the plane z = 4.

3. i) If $\bar{F} = (2x^2 - 3z)\,\bar{i} - 2xy\,\bar{j} - 4x\,\bar{k}$
Evaluate $\iiint_V (\nabla \cdot \bar{F})\, dv$

ii) Evaluate $\iiint_V \bar{F}\, dv$ where $\bar{F} = xy\,\bar{i} - 3x\,\bar{j} + \bar{k}$ and V is the region given by $x^2+y^2+z^2 = 4$, $x \geq 0$, $y \geq 0$, $z \geq 0$.

Practical No. 14(A)
Green's Divergence and Stoke's Theorms.

1. Verify Green's Theorem in the plane for $\oint_C (y - \sin x)\,dx + \cos x\, dy$, where C is the perimeter of the triangle with vertices (0,0), (π/2,0) and (π/2,1) .

2. Using Green's Theorem ,show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\oint (x\,dy - y\,dx)$. Hence find the area of the ellipse $s = a\cos\theta$, $y = b\sin\theta$.

3. Verify the divergence theorem for $\bar{F} = (x^2 - yz)\,\bar{i} + (y^2 = zx)\,\bar{j} + (z^2 - xy)\,\bar{k}$ for the cuboid $0 \leq x \leq a$, $0 \leq y \leq b$, $0 \leq z \leq c$

4. Using Gauss's divergence Theorem, Evaluate $\iint_S \bar{f} \cdot \bar{n}\, ds$, were $\bar{f} = 4x\,\bar{i} - 2y^2\,\bar{j} + z^2\,\bar{k}$ and S is the surface given by $x^2+y^2 = 4$, $z = 0$ and $z = 3$.

5. Verify stoke's Theorem for $\bar{f} = (x+y)\,\bar{i} + (2x-z)\,\bar{j} + (y+z)\,\bar{k}$ over the surface of the triangle cut off from the plane $3x+2y+z = 6$ by the co-ordinate planes.

6. Evaluate $\oint_C \bar{f} \cdot d\,\bar{r}$ by Stoke's Theorem where $\bar{f} = y^2\,\bar{i} + x^2\,\bar{j} - (x+z)\,\bar{k}$ And C is the boundary of the triangle with vertices at (0,0,0), (1,0,0), (1,1,0).

Practical NO. 15(A)
Revision

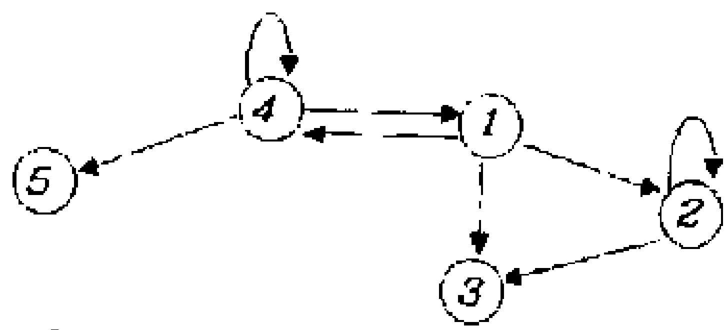
Practical No. 11 (B)
Relations and Digraphs

1. a) Compute the relation R on A={a,b,c,d,e} whose matrix of relation is

$$M_R = \begin{bmatrix} 1&1&0&0&0 \\ 0&0&1&1&0 \\ 0&0&0&1&1 \\ 0&1&1&0&0 \\ 1&0&0&0&0 \end{bmatrix}$$

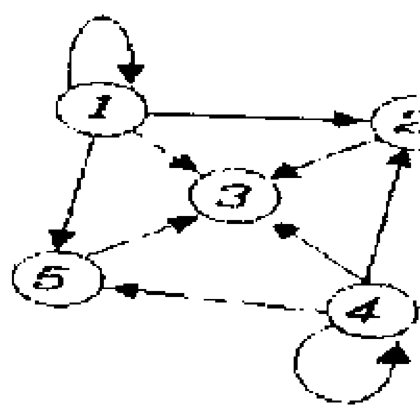
Also draw the digraph of this relation R and list indgree and outdegree of each vertex.

b) Find the relation determined by the digraph given below and give its corresponding matrix.

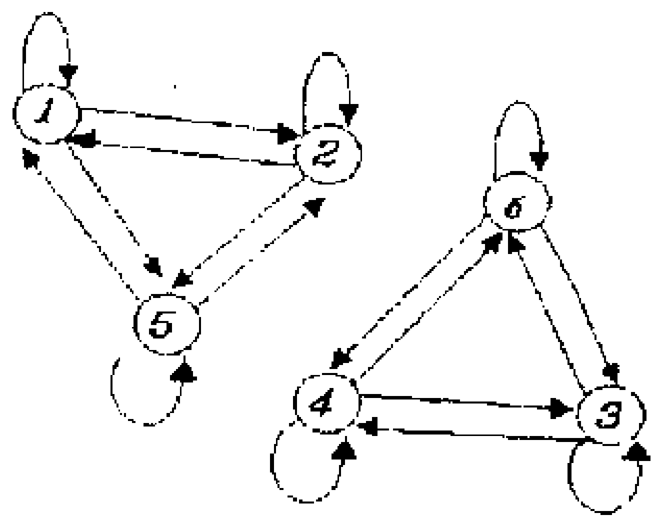


c) Let A ={a,b,c,d,e,} and relation R is given by set {(a,a), (a,b), (b,c) (c,e), (c,d), (d,e)} Compute R^2 and R^∞

2. a) Determine whether the relation R, whose digraph is given below is reflexive, irreflexive, symmetric, asymmetric, antisymmetric or transitive.



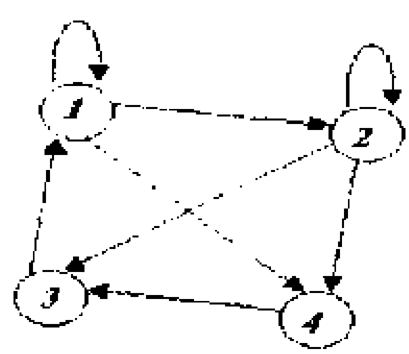
b) Determine whether the relation ; whose digraph is given below is an equivalence relation .



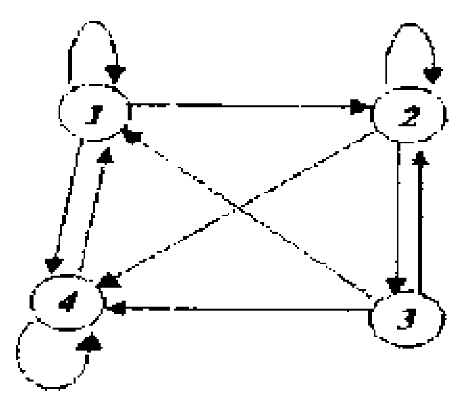
c) Let A={1 , 2 , 3 , 4} and
R={(1 , 1),(1 , 2),(2 , 1),(2 , 2),(3 , 4),(4 , 3),(3, 3),(4 , 4)}
Show that R is an eqivalence relation on A. Hence compute A/R

3 a) let A={2,3,6,12 } and let R and S be following relations on A such that xRy, if and only if 2/ x-y and xSy if and only if 3/x-y compute $\overline{R}$, R∩S. RUS and S^{-1}.

b) Let R and S be two relations whose corresponding digraphs are given by



digraph for R



digraph for S

compute $\overline{R}$, R∩S, RUS and S^{-1} .

4 a) Let A = B = {1,2,3,4,}
and R={ (1 , 1) , (1 , 3) , (2 ,3) , (3 , 1) , (4 , 2) , (4 , 4) }
S={(1,2),(2,3),(3,1),(3,2),(4,3)}
Compute $M_{R\cap S}$, $M_{R\cup S}$ and M_R^{-1} and $M_{\overline{S}}$

b) Find reflexive closure and the symmetric closure of R as well as S in (a)

c) Let A = { 1, 2, 3, 4 } and B = { 1, 2, 3 }. Let R and S be relations from A to B whose matrices are given by

$$M_S = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

5. a) Let A={1,2,3,4,} and R={(1,1),(1,4),(2,2),(2,3),(3,2), (3,3)(4,1),(4,4)} relation on A

i) Find transitive closure of R

ii) Compute the transitive closure of R by using the formula

$$M_R^{\infty} = M_R \vee (M_R)^2_{\odot} \vee (M_R)^3_{\odot}$$

where symbols have theire usual meanings.

b) Using Warshall's algoritm , find thr matrix of transitive closure for the relation R on A={1,2,3,4} where matrix for R is

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Practical No 12 (B)

Rooted and Labeled trees, Tree searching and Undirected trees

1 Let A={v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}}. and

$$V = \left\{ \begin{array}{l} (v_0, v_1), (v_0, v_2), (v_0, v_3), (v_1, v_4), (v_4, v_{11}), (v_4, v_{10}), (v_2, v_5), (v_2, v_6), \\ (v_6, v_{12}), (v_3, v_7), (v_7, v_8), (v_3, v_9), (v_8, v_{13}), (v_8, v_{14}), (v_{13}, v_{15}) \end{array} \right\}$$

is a relation on A

a) Show that T is a rooted tree with root v_0

b) List all level-3 vertices and list all leaves.

c) What are siblings of v_8 and descendent of v_8 ?

d) What is the height of (T, v_0) ?

e) Compute $T(v_3)$ and find height of it.

2. a) Construct the trees of the algebric express given below .

i) $(11 - (11 \times (11 + 11))) + (11 + (11 + 11))$

ii) $(3 - (2 - (11 - (9 - 4)))) \div (2 + (3 + (4 + 7)))$

iii) $(x \div y) \div ((x * 3) - (z \div 4))$

b) Represent the positional binary tree given by algebric expression.

$(3 - (2 * x)) + ((x - 2) - (3 + x))$ as a doubly linked list in the symbollic form.

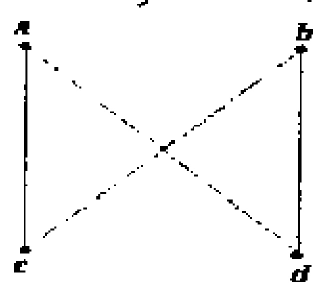
3 Draw a binary tree whose

a) Post order search produces

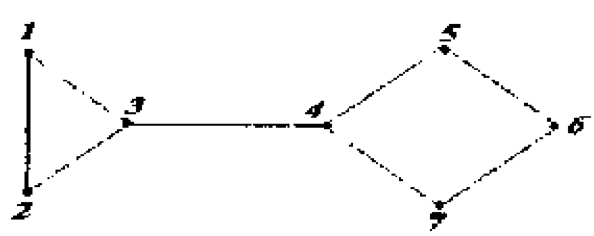
i) SEARCHING ii) TREEHOUSE.

b) Preorder search produces the string JBACDIHEGF

4 . Apply Prim's algorithm to the symmetric relation; whose graph is-



5 . Use Prim's algorithm to construct a spanning tree for the connected graph given below.



Use vertex 4 as the root of tree and draw the digraph of the spanning tree produced.

Practical No. 13 (B)
Language And Finite State Machines

1 Describe Precisely the language, L(G), produced by the grammar G that is describe all syntactically correct sentence in each case of the following.

i) $G = (V, S, v_0, \mapsto)$ where
$V = \{v_0, v_1, v_2, x, y, z\}$ and $S = \{x, y, z\}$ and
$\mapsto$: $v_0 \mapsto v_0v_1$, $v_0v_1 \mapsto v_2v_0$, $v_2v_0 \mapsto xy$;
$v_2 \mapsto x$ and $v_1 \mapsto z$

ii) $G = (V, S, v_0, \mapsto)$ where $V = \{v_0, x, y, z\}$ and $S = \{x, y, z\}$
and $\mapsto$: $v_0 \mapsto xv_0$, $v_0 \mapsto yv_0$, $v_0 \mapsto z$

iii) $G = (V, S, v_0, \mapsto)$ where $V = \{v_0, a\}$ & $S = \{a\}$
$\mapsto$: $v_0 \mapsto aav_0$ and $v_0 \mapsto aa$

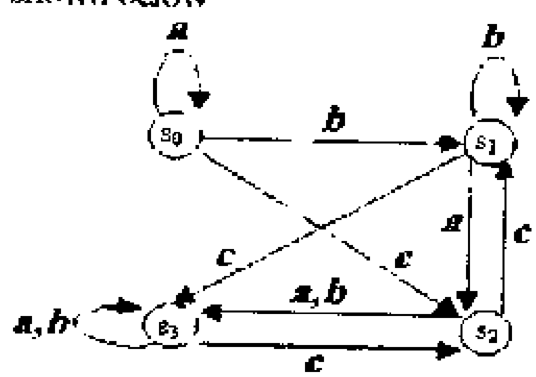
a) In each grammar G above, state whether the grammar is type 1, 2 or 3.
b) Draw the derivation tree for the string a^7 in the grammar G in (iii) above.
c) Give two distinct derivations (sequence of substitutions that start at v_0) for the string xyz $\in$ L(G), where G is grammar in (i) above.

2 a) Draw the diagraph of the machine whose state transition table is

	a	b	c
s_0	s_0	s_1	s_2
s_1	s_2	s_1	s_1
s_2	s_1	s_1	s_2
s_3	s_2	s_0	s_1

and label the edges with appropriate inputs.

b) Construct the state transition table of the finite state machine whose digraph is shown below



c) Let I = { 0,1} and S = { a , b } , Construct all possible state transition table of finite state machines that have S as state and I as input set.

3 a) Consider the machine whose state transition table is

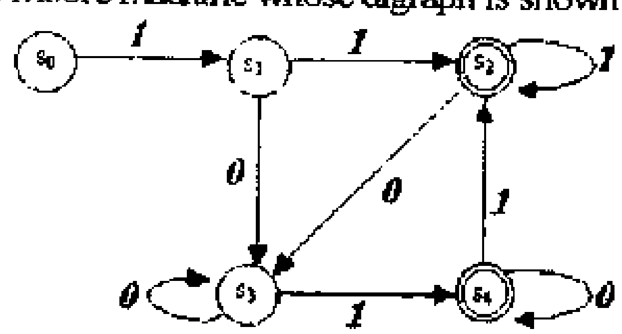
	0	1
1	1	4
2	3	2
3	2	3
4	4	1

and S = { 1 , 2 , 3 , 4 }

i) Show that R={ (1,1)(1,4)(4,1) (4,4)(2,2) (2,3) (3,2) (3,3) } is machine congruence.

ii) Construct the state transition table for the corresponding quotient machine.

b) Consider the Moore machine whose digraph is shown below

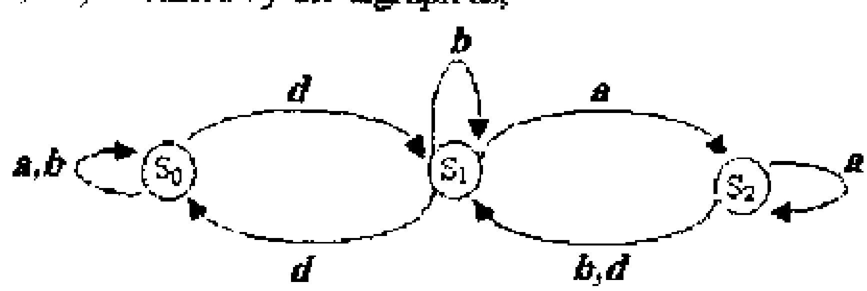


Show that the relation R on S is a machine congruence where matrix of relation R on S is,

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Draw the digraph of the corresponding quotient Moore machine

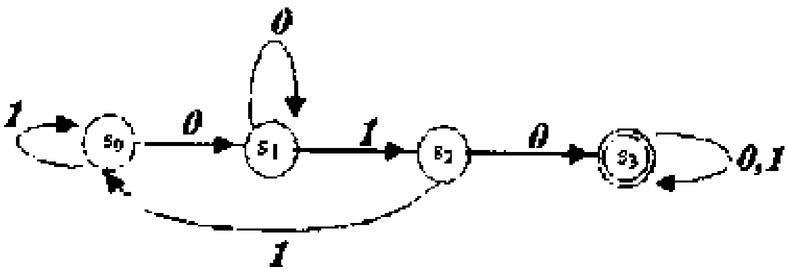
4. Let S={s_0, s_1, s_2 } and I ={a , b , d }.Consider the finite state machine M = (S, I, $\mathcal{F}$) defined by the digraph as,



Compute the f_{bad} , f_{add} , f_{badadd} and verify that

$f_{bad} \circ f_{add} = f_{badadd}$

5 Describe (in words) the language accepted by the Moore machine whose digraph is given by,



b) Describe (in words) the language accepted by the Moore machine whose state table is given by,

	0	1
S_0	S_1	S_0
S_1	S	S_2
S_2	S_1	S_0

The starting state is S_0 and set T of acceptance state is { S_2 }

Practical No 14(b)
Group Code

1 a) Consider the (3, 9) encoding function $e: B^3 \rightarrow B^9$ defined by

e (0, 0, 0) = 000000000 ; e (0, 0, 1) = 001001001;
e (0, 1, 0) = 010010010 ; e (0, 1, 1) = 011011011,
e (1, 0, 0) = 100100100 ; e (1, 0, 1) = 101101101;
e (1, 1, 0) = 110110110 ; e (1, 1, 1) = 111111111.

1) Find the minimum distance of e
2) How many errors will 'e' detect ?

b) Show that the (3, 7) encoding function $e: B^3 \rightarrow B^7$ is defined by ;

e (0, 0, 0) = 0000000 , e (0, 0, 1) = 0010110;
e (0, 1, 0) = 0101000 ; e (0, 1, 1) = 0111110;
e (1, 0, 0) = 1000101 ; e (1, 0, 1) = 1010011;
e (1, 1, 0) = 1101101 , e (1, 1, 1) = 1111011.

is a group code. Also find the minimum distance of this group code.

2. a) Let

$$H=\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

be a parity check matrix. Determine (2 , 5) group code function $e_H : B^2 \rightarrow B^5$

b) Compute

i)

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \oplus \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

ii)

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \star \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

3. Let d be the (6 , 5) decoding function defined by letting m be 5 in $d : B^{m+1} \rightarrow B^m$ defined by

$d(y) = y_1 , y_2 - - - - - y_m$ if $y = y_1 y_2 \text{------} y_m\, y_{m+1} \in B^{m+1}$. Determine d(y) for the word y in B^6

(a) y = 001101
(b) y = 110100

4. Let d be the (9 , 3) decoding function where $d : B^9 \rightarrow B^3$ defined by $d (y) = z_1 , z_2 , z_3$

$$\text{where } Z_i = \begin{cases} 1 \text{ if } (y_i , y_{i+3} , y_{i+6}) \text{ has atleast two 1's} \\ 0 \text{ if } (y_i , y_{i+3} , y_{i+6}) \text{ has less than two 1's.} \end{cases}$$

Determine d(y) for the word y in B^9 where

(a) y = 101111101
(b) y = 100111100

5 Let

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

be a parity check matrix. Decode the following words relative to a maximum likelihood decoding function associated with e_H

(a) 011001
(b) 101011

Practical No. 15(B)
Revision

Practical NO. 16
Groups

1. Show that set of all non-zero complex numbers forms an abelian group under usual multiplication.

2. Let $G = \{(a, b) \mid a, b \in Q, a \neq 0\}$. Show that $(G, *)$ is a group, where $(a, b) * (c, d) = (ac, ad + b)$. Is G abelian? Why?

3. Let G be a group and $(ab)^n = a^n b^n$ for three consecutive integers n, $\forall\, a, b \in G$. Show that G is an abelian group.

4. Prove that the set of matrices of the type $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

where $\theta \in R$, forms a group under matrix multiplication.

5. For $a, b \in Q$, define $a * b = a + b - ab$.
   Is $(Q, *)$ a group? Justify.

6. If in a group G, $a^5 = e$, $a b a^{-1} = b^2$, $a, b \in G$ then show that $O(b) = 31$

Practical No. 17
Subgroups

1. Let G be a group. Show that $H = \{x \in G \mid x a = a x, \forall a \in G\}$ is a subgroup of G

2. Let A, B be subgroups of a finite group G whose orders are relatively prime. Show that $A \cap B = \{e\}$.

3. Let $G = \{1,2,3,4,5,6,7,8,9,10\}$. Show that G is a cyclic group under multiplication modulo 11. Find all its generators, all its subgroups and order of every element. Also verify the Lagrange's theorem.

4. Prove that every proper subgroup of a group of order 35 is cyclic.

5. Let G be a group of order 17. Show that for any $a \in G$, either $O(a) = 1$ or $O(a) = 17$.

6. Let G be the group of all non-zero complex numbers under multiplication. Show that $H = \{a + bi \in G \mid a^2 + b^2 = 1\}$ is a subgroup of G.

7. Using Fermat's theorem, find the remainder
   when i) 9^{87} is divisible by 13
   ii) $5^{41} + 41^{12}$ is divisible by 11.

Practical NO. 18
Homomorphism

1. Show that the mapping $f: C \to R$ defined by $f(x + iy) = x$ is a homomorphism on to the additive group of complex numbers on to the additive group of real numbers. Find the kernel of f.

2. Let f be mapping from $(Z, +)$, the group of integers, to the group $G = \{1, -1\}$ under multiplication, defied as

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is even} \\ -1, & \text{if } x \text{ is odd} \end{cases}$$

Show that f is onto group homomorphism, Is f ono-one? Why?

3. Let G be the additive group of real numbers and G' be the multiplicative group of real numbers. Show that $\phi : G' \to G$ defined by $\phi(x) = \int_1^x (1/t)\, dt$, $\forall\ x \in G$, is a group homomorphism and find its kernel.

4. Let G= {1, -1, i, -i } be a group under multiplication and (Z,+) the group of integers under addition . Show that f: Z→G defined by $f(n) = i^n$ is a homomorphism. Find ker f. Is f onto? Why?

5. Let $G = \left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} a, b \in R \text{ and } a^2+b^2 \neq 0 \right\}$ be a group under multiplication of matrices and G', the group of non-zero complex numbers under multiplication. Show that f: G'→ G defined by $f(a + ib) = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ is an isomorphism.

Practical No. 19
Rings

1. Show that $R = \{a + ib \mid a, b \in Z, i = \sqrt{-1}\}$ is an integral domain with respect to addition and multiplication of complex numbers. Is it a field ? Justify.

2. Show that the set $\{a + b\sqrt{7} \mid a, b \in Q\}$ form a field under usual addition and multiplication.

3. For a, b ∈ Z ,define $a \oplus b = a + b - 1$ and $a \odot b = a + b - ab$, show that $(Z, \oplus, \odot)$ is a commutative ring with identity element.

4. In the ring $(Z_{10}, +_{10}, x_{10})$, find
   i) the zero elements of the ring
   ii) units in Z_{10}
   iii) additive inverse of 3
   iv) multiplicative inverse of 3
   v) $(-8)\ x_{10}\ 3$
   vi) $4 +_{10} (-8)$

Practical No. 20
Revision