NORTH MAHARASHTRA UNIVERSITY JALGAON.

Syllabus for T.Y.B.Sc. (Mathematics)
With effect from June 2014. (Semester system).

The pattern of examination of theory papers is semester system. Each theory course is of 50 marks (40 marks external and 10 marks internal) and practical course is of 100 marks (80 marks external and 20 marks internal). The examination of theory courses will be conducted at the end of each semester and examination of practical course will be conducted at the end of the academic year.

STRUCTURE OF COURSES

Semester –I		Semester- II	
MTH-351:	Metric Spaces	MTH-361:	Vector Calculus
MTH-352:	Real Analysis-I	MTH-362:	Real Analysis-II
MTH-353:	Abstract Algebra	MTH-363 :	Linear Algebra.
MTH-354:	Dynamics	MTH-364 :	Differential Equations
MTH-355(A):	Industrial Mathematics	MTH-365(A):	Operation Research
	OR		OR
MTH-355(B):	Number Theory	MTH-365(B):	Combinatorics
MTH-356(A):	Programming in C	MTH-366(A):	Applied Numerical Methods
	OR		OR
MTH-356(B):	Lattice Theory	MTH-366(B):	Differential Geometry

MTH-307: Practical Course based on MTH-351, MTH-352, MTH-361, MTH-362
MTH-308: Practical Course based on MTH-353, MTH-354, MTH-363, MTH-364
MTH-309: Practical Course based on MTH-355, MTH-356, MTH-365, MTH-366

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N.B. : Work load should not be increased by choosing optional courses.

SEMESTER – I
MTH-351: Metric Spaces

Unit -1: Metric Spaces **Periods-15, Marks-10**

1.1 Equivalence and countability ,
1.2 Metric Spaces
1.3 Limits in Metric Spaces.

Unit-2: Continuous functions on Metric Spaces. **Periods-15, Marks-10**

2.1 Reformulation of definition of continuity in metric spaces.
2.2 Continuous functions on metric spaces
2.3 Open sets
2.4 Closed sets
2.5 Homeomorphism

Unit-3: Connectedness and completeness of metric spaces. **Periods-15, Marks-10**

3.1 More about sets
3.2 Connected sets
3.3 Bounded and totally bounded sets
3.4 Complete metric spaces
3.5 Contraction mapping on metric spaces.

Unit-4 : Compactness of metric spaces. **Periods-15, Marks-10**

4.1 Compact metric space
4.2 Continuous functions on compact metric spaces
4.3 Continuity of the inverse function
4.4 Uniform continuity.

Recommended Book : Methods of Real Analysis by R.R. Goldberg.

Chapter I : 1.5, 1.6 Chapter IV : 4.2, 4.3
Chapter V : 5.2, 5.3, 5.4, 5.5 Chapter VI : 6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.7, 6.8.

Reference Book:

1. Metric spaces by E. T. Copson (Cambridge University Press)
2. Mathematical Analysis by S.C. Malik and Savita Arora.
3. A first course in Mathematical Analysis by D. Somsundaram & B. Chaudhari(Norosa Publishing House)

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MTH-352: Real Analysis – I

Unit 1: Riemann Integration **Periods-15, Marks-10**

1.1. Definition and existence of the integral .The meaning of $\int_a^b f(x)\,dx$ when $a \le b$. Inequalities for integrals.
1.2. Refinement of partitions
1.3. Darboux theorem (without proof)
1.4. Condition of integrability
1.5. Integrability of the sum and difference of integrable functions
1.6. The integral as a limit of a sum (Riemann sums) and the limit of a sum as the integral and its applications
1.7. Some integrable functions.

Unit 2: Mean value theorems of integral calculus **Periods-15, Marks-10**

2.1. Integration and Differentiation (the primitive)
2.2. Fundamental theorems of integral calculus
2.3. The first mean value theorem
2.4. The generalized first mean value theorem
2.5. Abel's lemma (without proof)
2.6. Second mean value theorem. Bonnets form and Karl Weierstrass form.

Unit 3: Improper integrals **Periods-15, Marks-10**

3.1. Integration of unbounded functions with finite limits of integration
3.2. Comparison test for convergence at a of $\int_a^b f(x)dx$
3.3. Convergence of improper integral $\int_a^b \frac{dx}{(x-a)^n}$
3.4. Cauchy's general test for convergence at a point a of $\int_a^b f(x)dx$
3.5. Absolute convergence of improper integral $\int_a^b f(x)dx$
3.6. Convergence of integral with infinite range of integration
3.7. Comparison test for convergence at ∞
3.8. Cauchy's general test for convergence at ∞
3.9. Convergence of $\int_a^\infty \frac{dx}{x^n}$, $a > 0$
3.10. Absolute convergence of improper integrals with infinite range of integration.
3.11. Abel's test and Dirichlet's test for convergence of $\int_a^\infty f(x)dx$. (Statements and examples only)

Unit 4: Legendre Polynomials **Periods-15, Marks-10**

4.1. Legendre's equation and its solution
4.2. Legendre's function of the first kind
4.3. Generating function for Legendre's polynomials
4.4. Trigonometric series for $p_n(x)$
4.5. Laplace's definite integrals for $p_n(x)$ (first and second integrals)

Recommended Books:

1) Mathematical Analysis by S. C. Malik and Savita Arora.
2) Ordinary and Partial Differential Equations by M. D. Raisinghania.

Reference Books:

1) Methods of Real Analysis by R. R. Goldberg.
2) Mathematical Analysis by S. K. Chatterjee.

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MTH-353: Abstract Algebra

Unit 1 : Normal Subgroup and Isomorphism Theorems for groups **Periods – 15, Marks – 10**

1.1 Normal subgroups
1.2 Quotient groups, Isomorphism theorems for groups
1.3 Isomorphism theorems for groups and examples
1.4 Generator of a subgroup
1.5 Commutator subgroup
1.6 Automorphism and inner automorphism

Unit 2 : Permutations **Periods – 15, Marks – 10**

2.1 Permutations
2.2 Cycles of permutation
2.3 Disjoint permutations
2.4 Permutation groups

Unit 3 : Quotient rings and Isomorphisms of rings **Periods – 15, Marks – 10**

3.1 Revision of Ring, integral domain, field, zero divisors, and basic properties
3.2 Characteristics of a ring
3.3 Subrings, ideals, left ideals, right ideals, principal ideals, prime ideals, maximal ideals.
3.4 Quotient rings
3.5 Field of quotients of an integral domain (Definition & Examples only)
3.6 Homomorphism of rings, Isomorphism theorems for rings.

Unit 4 : Polynomial Rings **Periods – 15, Marks – 10**

4.1 Definition of a polynomial ring, Properties of polynomial rings
4.2 Division Algorithm
4.3 Reducible and Irreducible polynomials
4.4 Eisenstein's Criterion.

Recommended Book :

1) A course in Abstract Algebra by V. K. Khanna and S. K. Bhambri, Vikas Publishing House Pvt. Ltd. New Delhi.
2) A first course in Abstract Algebra by J. B. Fraligh.

Reference Books :

1) University Algebra by N. S. Gopalkrishnan.

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MTH-354: Dynamics

Unit 1: Kinematics **Periods – 15, Marks – 10**

1.1 Displacement
1.2 Motion in a straight line velocity and acceleration
1.3 Motion in a plane velocity and acceleration.
1.4 Radial and transverse component of velocity and acceleration
1.5 Angular velocity and acceleration
1.6 Tangential and normal components of velocity and acceleration.

Unit 2 : Rectilinear motion **Periods – 15, Marks – 10**

2.1 Motion in a straight line with constant acceleration
2.2 Motion of a train between two stations
2.3 Simple harmonic motion
2.4 Hook's law (a) Horizontal elastic strings (b) Vertical elastic strings.

Unit 3 : Uniplanar motion **Periods – 15, Marks – 10**

3.1 Projectile – Introduction
3.2 Projectile – Equation of trajectory
3.3 Projection to pass through a given point
3.4 Envelope of the paths.

Unit 4 : Central forces **Periods – 15, Marks – 10**

4.1 Motion of a particle under central force
4.2 Use of pedal co-ordinates and equation
4.3 Apses.

Recommended Book :

A text book on Dynamics by M. Ray. (S. Chand and Company, New Delhi).

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MTH-355(A): Industrial Mathematics

Unit- 1 : Statistical methods **Periods -15, Marks – 10**

1.1 Sample space, events, Types of events, Random variable, Distribution function, Discrete random variable, Probability mass function, Properties.

1.2 Continuous random variable, Probability density function, Various measures of central tendency, Dispersion, Skewness and kurtosis for continuous distribution function and its properties.

1.3 Bernoulli distribution, Binomial distribution, Poission distribution, pmf, its mean, variance and important properties, Simple examples.

1.4 Standard continuous probability distribution, Exponential distribution, its pdf mean, variance.

Unit- 2 : Statistical Quality Control (S.Q.C.) **Periods – 15, Marks – 10**

2.1 Introduction to sampling methods, Simple random sampling, random sampling, Chance and assignable causes of variation, Use of S.Q.C.

2.2 Process and product control, Control charts, 3 - σ (Three sigma) control limits.

2.3 Tools for statistical quality control, Control chart for variables.

2.4 Control chart for mean or x chart, control chart for range (R chart) and their interpretations.

2.5 Control chart for standard deviation or σ - chart. Control chart for attributes.

2.6 Control chart for fraction defective or p – chart, p – chart for variable sample sizes.

2.7 Control chart for number of defectives or np – charts. Control chart for number of defectives per unit (c – chart). Use of c – chart.

Unit -3: Queuing Theory **Periods – 15, Marks – 10**

3.1 Applications of queuing models. 3.2 Introduction, characteristics of queuing models.

3.3 Waiting time and Idle time cost. 3.4 Transient and steady states of the system.

3.5 Single-channel Queuing theory. 3.6 Multi-channel Queuing theory.

3.7 Monte Carlo Technique supplied to Queuing problems.

Unit -4 : Sequencing and project scheduling by PERT (Programme Evaluation and Review Technique) and CPM (Critical Path Method) **Periods – 15, Marks – 10**

4.1 Examples on application of sequencing models 4.2 Sequencing problems

4.3 Processing each of n – jobs through m – machines 4.4 Processing n – jobs through two machines

4.5 Processing each of n – jobs through three machines 4.6 Approaches to more complex sequencing problems.

4.7 Routing problems in networks. The traveling salesman problem. Minimal path problem.

4.8 Basic steps in PERT and CPM : Introduction, Historical background, phases of project scheduling work break down structure, Network logic, Numbering the events (Fulkerson's rule), Measure of activity, Frequency distribution curve for PERT, Examples.

4.9 PERT computations : Forward pass computations, Backward pass computations, Computations in tabular form, Slack, Critical path.

Recommended Books :

1) Fundamentals of Mathematical Statistics by S. C. Gupta and V. K. Kapoor. S. Chand and Sons Pub. Eigth Edition 1991. Unit 1 – Chapter 5 : 5.1, 5.2, 5.3, 5.4. Chapter 7 : 7.1, 7.1.1, 7.2, 7.2.1, 7.3, 7.3.1, 7.3.2, 8.6.

2) Fundamentals of Statistics by S. C. Gupta. Sixth revised and enlarged edition. Himalaya Pub. House. Unit 2 – Chapter 21.

3) Operation Research by R. K. Gupta and D. S. Hira. S. Chand and Comp. Ltd. Unit 3 and Unit 4 – Chapter 5, 7, 10, 11.

Reference Books :

1) Applied Statistics, by S. C. Gupta and V. K. Kapoor.

2) Mathematical Statistics by J. N. Kapoor and H. C. Saxena.

3) Statistical Quality Control by M. Mahajan.

4) Statistical Quality Control by L. Grant.

5) Mathematical Models in Operation Research by J. K. Sharma (Tata Macgrahill)

6) Quality Control and Industrial Statistics by Acheson and J. Duncan.

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MTH-355(B): Number Theory

Unit 1 : Divisibility Theory **Periods – 15, Marks – 10**

1.1 The Division Algorithm
1.2 The Greatest Common Divisor
1.3 The Euclidean Algorithm
1.4 The Diophantine Equation $ax + by = c$

Unit 2 : Primes and Their Distribution **Periods – 15, Marks – 10**

2.1 The Fundamental Theorem of Arithmetic
2.1 The Sieve of Eratosthenes
2.3 The Goldbach Conjecture

Unit 3 : The theory of congruences and Fermat's theorem **Periods – 15, Marks – 10**

3.1 Karl Friedrich Gauss
3.2 Basic Properties of Congruence
3.3 Special Divisibility Tests.
3.4 Linear Congruences.
3.5 Pierre de Fermat
3.6 Fermat's Factorisation Method
3.7 The Little Theorem
3.8 Wilson Theorem

Unit 4 : Perfect numbers and Fibonacci numbers **Periods – 15, Marks – 10**

4.1 The Search for perfect Numbers
4.2 Mersenne Numbers
4.3 Fermat's Numbers
4.4 The Fibonacci Sequence
4.5 Certain Indentities Involving Fibonacci Numbers.

Recommended Book :

1) Elementary Number Theory (Second Edition) - David M. Burton. (Universal Book Stall, New Delhi)

Reference Books :

1) Introduction to Analytic Number Theory - T. M. Apostol (Springer International student Edition)
2) Number Theory – Hari Kishan, Krishna Prakashan Media (p) Ltd, Meerat.

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MTH-356(A): Programming in C

Unit 1: Basic concepts **Periods – 15 , Marks - 10**

1.1 Introduction
1.2 Character set
1.3 C tokens, keywords
1.4 Constants
1.5 variables, data types
1.6 variables, symbolic constants
1.7 over flow, under flow
1.8 operators of arithmetic, relational, logical, assignment , increment and decrement, conditional and special type

Unit 2: Expressions and conditional statements **Periods – 15 , Marks – 10**

2.1 Arithmetic expression and its evaluation precedence of arithmetic operators type
2.2 Conversion, operator precedence, mathematical functions
2.3 Reading and writing a character
2.4 Formatted input and out put
2.5 Decision making , if, is-else, else-if, switch and go to statements

Unit 3: Loops: Decision making and Looping **Periods – 15 , Marks – 10**

3.1 Sentinel loops. While loop, do-while loop and for statements
3.2 Jump in loops, continue, break and exit statements

Unit 4: Arrays and Functions **Periods – 15, Marks – 10**

4.1 One dimensional, two dimensional and multidimensional arrays. Declaration and initialization of arrays
4.2 Need for user defined functions, multi-function program
4.3 Elements of function, definition of functions, return values and their types
4.4 Function calls, function declaration, category of functions, functions that return multiple values.

Recommended Book:

1) Programming in ANSI C by E. Balagurusamy, Mcgraw-Hill company.

Reference Book :

1) LET Us C by Yashwant Kanitkar.

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MTH-356(B): Lattice Theory

Unit 1: Posets **Periods – 15 , Marks - 10**

1.1 Posets and chains
1.2 Digrammatical representation of poset, Hasse diagram
1.3 Maximal and minimal elements of the subset of a poset, Zorn's lemma(Statement only)
1.4 Supremum and infimum of the subset of a poset
1.5 Poset isomorphism
1.6 Duality principle.

Unit 2 : Lattices **Periods – 15 , Marks - 10**

2.1 Two definitions of a lattice and equivalence of two definitions.
2.2 Modular and distributive inequalities in a lattice.
2.3 Sublattice and semilattice.
2.4 Complete lattice.

Unit 3 : Ideals and homomorphisms **Periods – 15 , Marks - 10**

3.1 Ideals, Union and intersection of ideals.
3.2 Prime ideals
3.3 Principal ideals
3.4 Dual ideals.
3.5 Principal dual ideals
3.6 Complements, Relative complements
3.7 Homomorphism, Join and meet homomorphism.

Unit 4 : Modular and Distributive lattices **Periods – 15 , Marks - 10**

4.1 Modular lattice
4.2 Distributive lattice
4.3 Sublattice of a modular lattice
4.4 Homomorphic image of modular lattice
4.5 Complemented and relatively complemented lattice.

Recommended Book:

1) Lattices and Boolean Algebra (First concepts) by Vijay K. Khanna (Vikas Publ. House, Pvt. Ltd.) Chapter - 2, 3, 4.

Reference Book:

1) Discrete Mathematics by Schaum outline series.

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SEMESTER – II
MTH-361: Vector Calculus

Unit 1: Vector Algebra **Periods-15, Marks-10**

1.1 Vectors, Scalars, laws of vector algebra, scalar field, vector field, dot product, cross, scalar triple product, vector triple product

1.2 Properties

1.3 Ordinary derivative of vectors, space curves, continuity and differentiability of vectors

1.4 Differentiation formulae

1.5 Partial differentiation of vectors, differentials of vectors.

Unit 2: The Vector Operator Del **Periods-15, Marks-10**

2.1 The vector differentiation operator del.

2.2 Gradient,.

2.3 Divergence and curl.

2.4 Formulae involving del. Invariance.

Unit 3: Vector Integration **Periods-15, Marks-10**

3.1 Ordinary integrals of vectors.

3.2 Line integrals.

3.3 Surface integrals.

3.4 Volume integrals

Unit 4: Integral Theorems **Periods-15, Marks-10**

4.1 Green's theorem in the plane.

4.2 Stokes theorem.

4.3. Gauss Divergence theorem and related integral theorems

4.4 Integral operator theorem for Del.

Recommended Books:

1) Vector Analysis By Murray R Spiegel , Schaum Series, Mcgraw Hill Book Company.

2) Vector Calculus By Shanti Narayan And P.K. Mittal, S. Chand & Co., New Delhi

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MTH-362: Real Analysis-II

Unit 1: Sequence of real numbers and Sequence of functions **Periods-15, Marks-10**

1.1. Definition of sequence and subsequence of real numbers. Convergence and divergence of sequence of real numbers
1.2. Monotone sequence of real numbers
1.3. Pointwise convergence of sequence of functions
1.4. Uniform convergence of sequence of functions
1.5. Cauchy's criteria for uniform convergence of sequence of functions
1.6. Consequences of uniform convergence

Unit 2: Series of real numbers **Periods-15, Marks-10**

2.1. Convergence and divergence
2.2. Series with non-negative terms
2.3. Alternating Series
2.4. Conditional convergence and absolute convergence
2.5. Rearrangement of series
2.6. Test for absolute convergence
2.7. Series whose terms form non-increasing sequence

Unit 3: Series of functions **Periods-15, Marks-10**

3.1. Pointwise convergence of series of functions
3.2. Uniform convergence of series of functions
3.3. Integration and differentiation of series of functions
3.4. Abel's Summability

Unit 4: Fourier series in the range $[-\pi, \pi]$ **Periods-15, Marks-10**

4.1. Fourier series and Fourier coefficients
4.2. Dirichlet's condition of convergence (statement only)
4.3. Fourier series for even and odd functions
4.4. Sine and Cosine Series in half range

Recommended Books:

1) Methods of Real Analysis by R.R. Goldberg.
   Unit 1:- 2.1, 2.3, 2.4, 2.6, 9.1, 9.2
   Unit 2:- 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7
   Unit 3:- 9.4, 9.5, 9.6
2) Laplace Transform and Fourier series, Schaum series (Unit – 4).

Reference Books:

1) Mathematical Analysis by S.C. Malik and Savita Arora.
2) Mathematical Analysis by S.K. Chatterjee.

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MTH-363: Linear Algebra

Unit 1 : Vector Spaces **Periods – 15, Marks – 10**

1.1 Vector spaces, Subspaces
1.2 Necessary and sufficient conditions for a subspace
1.3 Addition, Intersection and union of subspaces
1.4 Quotient space
1.5 Linear span and properties
1.6 Linear dependence and independence.

Unit 2 : Basis and Dimensions **Periods – 15, Marks – 10**

2.1 Basis and dimension of finite dimensional vector spaces
2.2 Co-ordinates of a vector
2.3 Existence theorem, Invariance of elements in a basis
2.4 Extension theorem
2.5 Theorems on basis and dimensions

Unit 3 : Linear Transformations **Periods – 15, Marks – 10**

3.1 Range space and null space of linear transformations
3.2 Rank and nullity theorem
3.3 Vector space L(V,W)
3.4 Algebra of linear transformations and theorems on isomorphism
3.5 Invertible linear transformations
3.6 Singular and non-singular linear transformations
3.7 Representation of linear transformation by matrix.

Unit 4 : Eigen values and Eigen vectors **Periods – 15, Marks – 10**

4.1 Matrix polynomial
4.2 Characteristics polynomial and minimum polynomial
4.3 Eigen values and Eigen vectors of linear transformation
4.4 Similarity
4.5 Diagonalization of matrix
4.6 Cayley Hamilton Theorem

Recommended Book:

1) Theory and Problems of Linear Algebra, by S. Lipschutz, Schaum's outline series, SI(Metric) edition, (1987), McGraw Hill Book Company.

Reference Books:

1) Theory and Problems of Linear Algebra, by S. Lipschutz, Schaum's outline series, edition, Tata McGraw Hill Edition 2005.
2) University Algebra by N. S. Gopalkrishnan
3) A course in Abstract Algebra by V. K. Khanna and S. K. Bhambri, Vikas Publishing House Pvt. Ltd. New Delhi.

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MTH-364: Differential Equations

Unit 1: Exact Differential Equations **Periods – 15, Marks – 10**

1.1 Definition, condition of exactness of a linear differential equation of order n

1.2 Working rule for solving exact equations, examples of type-I

1.3 Integrating factor, examples of type-II

1.4 Exactness of non-linear equations, Solutions by trial, examples of type -III

1.5 Equations of the form $\frac{d^n y}{dx^n}=f(x)$, examples on type-IV

1.6 Equations of the form $\frac{d^2 y}{dx^2}=f(y)$, examples on type-V.

Unit 2: Linear Differential Equations of Second Order **Periods – 15, Marks – 10**

2.1The standard form of linear diff. eqn. of second order

2.2 Complete solution in terms of one known integrals belonging to C.F.

2.3 Rule for getting an integral belong to C.F., working rule for finding complete solution when an integral of C.F. is known.

2.4 Removal of first derivative, reduction to normal form, working rule for solving problems by using normal form

2.5 Transformation of the equation by changing the independent variable, working rule.

Unit 3: Power Series Method **Periods – 15, Marks – 10**

3.1 Introduction, some basic definitions, ordinary and singular points

3.2 Power series solution, series solution about regular singular point x=0, Frobenius method

3.3 Frobenius method type-I

3.4 Frobenius method type-II

3.5 Frobenius method type-III

3.6 Frobenius method type-IV based on.

Unit 4: Difference Equations **Periods – 15, Marks – 10**

4.1 Introduction, order of difference equation, degree

4.2 Solution and formation

4.3 Linear difference equations, linear homogenous difference equations with constant coefficients

4.4 Non-homogenous linear difference equations.

Recommended Books:

1) Ordinary And Partial Differential Equations By M.D. Raisinghania, S. Chand & Co.

2) Numerical Methods By V.N. Vedamurthy And N.Ch.S.N. Iyengar, Vikas Publishing House.

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MTH-365(A): Operation Research

Unit 1 : Linear programming problem (LPP) **Periods – 15, Marks – 10**

1.1 Formation of LPP
1.2 Solution of LPP by graphical method
1.3 Solution of LPP by simplex method
1.4 Artificial variable technique (Big M method)
1.5 Special cases in LPP : a) Unbounded solution b) Alternate solution c) No solution by graphical as well as simplex method
1.6 Degeneracy in simplex method and its resolution.

Unit 2 : Transportation problem (TP) **Periods – 15, Marks – 10**

2.1 Formation of TP. TP as LPP
2.2 Methods for finding IBFS : a) North –West corner rule. b) Matrix minima method (Least cost method) c) Vogel's approximation method (VAM)
2.3 Optimality test and optimization of solution to TP by U-V method (MODI).
2.4 Special cases in TP : a) Alternate solution b) Maximization of TP c) Degeneracy in solving TP. d) Restricted transportation problems.

Unit 3 : Assignment problem (AP) **Periods – 15, Marks – 10**

3.1 Formation of Assignment problem AP as TP
3.2 Hungerian method for solving AP
3.3 Special cases in AP: a) Alternate solution b) Maximization of AP c) Restricted AP.

Unit 4: Simulation **Periods – 15, Marks – 10**

4.1 Introduction
4.2 Reasons for using simulation
4.3 Methodology for simulation
4.4 Advantages and disadvantages of simulation
4.5 Some typical applications
4.6 Monte-Carto simulation, Random number generation
4.7 Simulation for an inventory system
4.8 Simulation of queuing system
4.9 Simulation of maintenance problem
4.10 Investment decision (or Capital budgeting) through simulation.

Recommended book:

1) OPERATION RESEARCH (Quantitative Techniques for Management) sultan chand & sons (Eighth edition. Reprinted in 2011)
Articles: 14.1, 14.3, 14.4, 14.5, 14.6, 14.7, 14.7.1, 14.7.2, 14.9, 14.10, 14.11, 14.12.

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MTH-365(B): Combinatorics

Unit 1 : Fundamental Principles of counting **Periods – 15, Marks – 10**

1.1 The rules of sum and product
1.2 Permutations
1.3 Combinations
1.4 Permutations and combinations with repetitions
1.5 The Binomial theorem : Pascal's identity, Vander Monde's identity, Multinomial theorem
1.6 Ramsey number
1.7 Catalan numbers
1.8 Stiriling numbers and Bell numbers.

Unit 2 : The principles of Inclusion and Exclusion **Periods – 15, Marks – 10**

2.1 The principles of Inclusion and Exclusion
2.2 Generalization of the principles of Inclusion and Exclusion
2.3 The Pegion-Hole principle
2.4 The generalized Pegion-Hole principle
2.5 Derangements.

Unit 3 : Generating Functions **Periods – 15, Marks – 10**

3.1 Generating Functions
3.2 Introductory examples
3.3 Partitions of integers
3.4 The exponential Generating Functions
3.5 The summation operator
3.6 Calculational Techniques : Geometric series, Use of partial fractions decomposition.

Unit 4 : Recurrence Relations **Periods – 15, Marks – 10**

4.1 The first order linear Recurrence Relations : Back traking method, Forward chaining method, Summation method.
4.2 The second order linear homogeneous Recurrence Relation with constant coefficients.
4.3 The non-homogeneous Recurrence Relations, Characteristic equation method.
4.4 The method of generating functions.

Recommended Book:

1) Theory and Problems of Combinatorics by C. Vasudeo (New Age International Pub.).

Reference Books:

1) Combinatorial Theory and Applications by V. Krishnamurti.
2) Introduction to Combinatorics by Richard R. I. Bernaldi (Publication Worth – Holland)

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MTH-366(A): Applied Numerical Methods

Unit 1: Simultaneous Linear equations **Periods - 15, Marks – 10**

1.1 Method of factorization or triangularization.
1.2 Crout's method
1.3 Inverse of a matrix by Crout's method.
1.4 Gauss- Seidal iteration method.
1.5 Relaxation Method

Unit 2: Interpolation with Unequal Intervals: **Periods - 15, Marks – 10**

2.1 Newton's Divided Difference Formula.
2.2 Lagrange's Interpolation Formula
2.3 Inverse Interpolation
2.4 Lagrange's Method
2.5 Iterative Method.

Unit 3: Numerical Differentiation and Integration: **Periods - 15, Marks – 10**

3.1 Numerical differentiation.
3.2 Derivatives using Newton's Forward Difference formulae.
3.3 Derivative using Newton's Backward Difference formulae.
3.4 Numerical integration, general quadrature formula.
3.5 Trapezoidal rule.
3.6 Simpson's 1/3 rule.
3.7 Romberg method.

Unit 4: Numerical Solutions of Ordinary Differential Equations **Periods - 15, Marks – 10**

4.1 Picard's Method
4.2 Taylor's series Method
4.3 Modified Euler's method.
4.4 Runge Kutta Fourth order Method.
4.5 Miline's Method .
4.6 Adam-Bashforth Method

Recommended Text Book:

1. Numerical Methods by V.N. Veda Murthy and N.Ch.S.N. Iyengar, Vikas publications, India.

Reference Book:

1. Numerical Analysis by S.S. Sastry.

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MTH-366(B): Differential Geometry

Unit -1: Curves in Spaces **Period 15, Marks 10**

Space curves; parametric and vector representation of curves, Tangent line, Osculating plane and Normal plane, Principal normal and binormal. The unit vectors , $\bar{t}$, $\bar{n}$, $\bar{b}$, rectifying plane, curvature and Torsion, Radius of curvature and radius of torsion, screw curvature Serret- Frenet formulae, curvature and torsion of a curve, helices.

Unit-2: Curvatures **Period 15, Marks 10**

The circle of curvature, osculating sphere, Involute and Evolute of space curve, The spherical Indicatrices (Spherical images), Bertrand curves.

Unit-3: Envelopes of Surfaces: **Period 15, Marks 10**

Parametric representation of surfaces. Tangent plane and normal line at a point on surface, first fundamental form or metric and second fundamental form, Envelope and characteristics and the edge of regression relating to one and two parameter family of surfaces.

Unit-4: Developable Surfaces: **Period 15, Marks 10**

Ruled surface (Developable and skew). Developable surface, Developable associated with space curves , characterization of a developable surface.

Recommended Book:

Differential Geometry by Dr. S. C. Mittal and D.C. Agrawal (Krishna Prakashan Mandir)

Chapter 1: 2, 5, 7, to 13, 15, 16, 18, 19, 20 Chapter 2 : 1, 3, 4, 5, 6, 7, 10 Chapter 3: 9 to 14

Reference Books:

1) Differential Geometry by M. L. Khanna (Jaiprakash Nath and Co.)
2) Differential Geometry by P.P. Gupta, G.S. Malik and S.K. Pundir (Pragati Prakashan)
3) Three dimensional Differential Geometry by Bansilal (Atmaram and Sons)

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NORTH MAHARASHTRA UNIVERSITY JALGAON

T.Y.B.Sc. Mathematics

Practical Course MTH-307

Based on MTH-351, MTH-352, MTH-361, MTH-362.

(With effect from June 2014)

Index

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2	Continuous Functions on Metric Spaces
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MTH-351: Metric Spaces

Practical No.-1: Metric Spaces

1. a) Show that if A and B are countable sets then, $A \times B$ is countable.
   b) Show that the intervals (0 , 1) and [0 , 1] are equivalent.
2. a) Show that if d is a metric for a set M then so also 2d.
   b) For points $x = (x_1, x_2)$ and $y = (y_1, y_2)$ in R^2 define $d(x, y) = |x_1 - y_1| + |x_2 - y_2|$. Show that d is a metric for R^2.
3. a) If $\{x_n\}_{n=1}^{\infty}$ is a convergent sequence in R_d then show that there exists positive integer N such that $x_N = x_{N+1} = x_{N+2} = \ldots\ldots\ldots$
   b) Show that a sequence of points in any metric space cannot converge to two distinct points.
4. Let l^1 be the class of all sequences $\{s_n\}_{n=1}^{\infty}$ of real number such that $\sum_{n=1}^{\infty} |s_n| < \infty$. Show that if $s = \{s_n\}_{n=1}^{\infty}$, and $t = \{t_n\}_{n=1}^{\infty}$, are in l^1 then $d(s, t) = \sum_{n=1}^{\infty} |s_n - t_n|$ defines a metric for l^1 .
5. a) If $\{x_n\}_{n=1}^{\infty}$ is a cauchy sequence of points in the metric space M and if $\{x_n\}_{n=1}^{\infty}$ has subsequence which converges to $x \in M$ then prove that $\{x_n\}_{n=1}^{\infty}$ itself is convergent to x.
   b) Let M = (0, 1) and d be a metric defined on M by $d(x, y) = |x - y|\ \forall\ x, y \in M$. Show that $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ in M is Cauchy but not convergent in M.

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Practical No.-2: Continuous Functions on Metric Spaces

1. a) i) Let M = [0,1] & d be absolute value metric for M. Find S (¼, ½).
      ii) Let $M = R_d$, the real line with discrete metric and if $a \in R_d$ then find S (a, 1) and S (a, 2).
   b) Let f and g be continuous functions on a metric space M and Let A be the set of all $x \in M$ such that $f(x) < g(x)$. Prove that A is open.
2. a) If A and B are subsets of a metric space M such that $A \subset B$ then prove that $\overline{A} \subset \overline{B}$.
   b) Give an example of a sequence $\{A_1, A_2, \ldots\}$ of nonempty closed subsets of R^1 such that both of the following conditions hold : (i) $A_1 \supset A_2 \supset A_3 \ldots$ (ii) $\bigcap_{n=1}^{\infty} A_n = \phi$.
3. a) Let M be a metric space and let $A \subset B \subset M$. If A is dense in B and if B is dense in M then prove that A is dense in M.
   b) Give an example of a set E such that both E and its complement are dense in R^1. Can E be closed?
4. a) Prove that $(0, \infty)$ with absolute value metric is homeomorphic to R^1.
   b) Prove that the metric spaces [0, 1] and [0, 7] with absolute value metric are homeomorphic.
5. Give an example of subsets A and B of R^2 such that all three of the following conditions hold
   i) Neither A nor B is open ii) $A \cap B = \phi$ iii) $A \cup B$ is open.

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Practical No.- 3: Connectedness and Completeness of Metric Spaces

1. a) Let A = [0, 1] be a metric space with absolute value metric d. Which of the following subsets of A are open subsets of A ?
   i) (½, 1] ii) (½, 1)
   b) Show thus [0, 1] with usual metric is always complete and connected.
2. a) Prove that [0,1] is not connected subset of R_d.
   b) Give an example to show that B is not connected though A and C are connected subsets of metric space M such that $A \subset B \subset C$.
3. Prove that R^2 is complete.
4. a) If $T(x) = x^2$ $(0 \leq x \leq 1/3)$ then prove that T is contraction on $[\,0, \frac{1}{3}\,]$.
   b) If T is contraction on metric space M then prove that T is continuous on M.
5. a) Give an example of a bounded subset of l^{∞} which is not totally bounded.
   b) Prove that every finite subset of metric space M is totally bounded.

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Practical No.- 4: Compact Metric Spaces

1. If A and B are compact subsets R^1 then prove that $\overline{A} \times \overline{B}$ is compact subset of R^2.
2. Give an example of i) a connected subset of R^1 that is not compact.
   ii) a compact subset of R^1 which is not connected.
3. If f is continuous function from the compact metric space M_1 into the metric space M_2 then prove that the range $f(M_1)$ is a bounded subset of M_2
4. a) Show that $f(x) = x^2$ $\forall\, x \in (-\infty, \infty)$ is not uniformly continuous on $(-\infty, \infty)$.
   b) Show that $f(x) = x^3$ $\forall\, x \in [\,0,1]$ is uniformly continuous on [0,1].
5. a) Show that if $f(x) = \frac{1}{1+x^2}$ for $-\infty < x < \infty$ then f attains a maximum value but does not attain minimum value.
   b) If $f : A \to R^1$ and f attains a maximum value at $a \in A$ then show that $f(a) = \operatorname{lub}_{x \in A} f(x)$

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MTH-352: Real Analysis-I

Practical No.- 5: Riemann Integration

1. Let f(x) = x^2 defined on [0, a].

   Find a) U(P ,f) b) L (P ,f) c) Show that f $\in$ R[0, a] and $\int_0^a f(x)dx = \frac{a^3}{3}$.

2. Let f(x) be a function defined on [0, 2] such that $f(x) = \begin{cases} 0 & \text{, when } x = \frac{n}{n+1} \text{ or } \frac{n+1}{n} \\ 1 & \text{, otherwise} \end{cases}$

   a) Is f integrable on [0, 2] ? If so evaluate $\int_0^2 f(x)dx$.

   b) Examine f for continuity at point x = 1.

3. A function defined on [0, 1] as $f(x) = \frac{1}{a^{r-1}}$ if $\frac{1}{a^r} < x \leq \frac{1}{a^{r-1}}$ where a is an integer greater than 2, and r = 1, 2, 3,…. Show that a) $\int_0^1 f(x)dx$ exist b) $\int_0^1 f(x)dx = \frac{a}{a+1}$.

4. Let the function f, defined as f(x) =$(-1)^{r-1}$ if $\frac{1}{r+1} < x \leq \frac{1}{r}$ where r = 1, 2, 3, … and f (0) = 0. Show that, a) f is integrable on [0,1] b) Evaluate $\int_0^1 f(x)dx$.

5. If $0 < a < b$ and p is a positive integer, then show that $\lim\limits_{n \to \infty} \sum\limits_{r=1}^{pn} \frac{1}{na + rb} = \frac{1}{b} \log\left(1 + \frac{pb}{a}\right)$

6. Evaluate $\lim\limits_{n \to \infty} \left[\frac{n^n}{n!}\right]^{1/n}$.

7. Show that $\int_1^2 f(x)dx = \frac{11}{2}$ where $f(x)$ = 3x+1 using Riemann definition of definite integral as limit of sum.

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Practical No.- 6: Mean Value Theorems

1. Show that $\frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5+3cosx} \leq \frac{\pi^3}{6}$.

2. If a > 0 then, show that $ae^{-a^2} < \int_0^{-a^2} e^{-x^2}\, dx < \tan^{-1} a$.

3. Show that $\frac{\pi}{4} \leq \int_0^{\pi} secxdx \leq \frac{\pi}{2\sqrt{2}}$.

4. Show that $\lim \int_0^1 \frac{nf(x)}{1+n^2x^2} dx = \frac{\pi f(0)}{2}$.

5. Verify Second Mean Value theorem for the function $f(x) = x$ and $g(x) = e^x$ defined in [0,1].

6. If 0< $a < b$ then show that a) $\left| \int_a^b sin(x^2)dx \right| \leq \frac{1}{a}$ b) $\left| \int_a^b \frac{\sin x}{x} dx \right| \leq \frac{2}{a}$.

7. Verify Weierstrass Second Mean Value theorem for the integral $\int_{\pi}^{2\pi}$ x sinx dx.

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Practical No.- 7: Improper Integrals

1. Show that

a) $\int_0^2 \frac{\log x}{\sqrt{2-x}} dx$ is convergent b) $\int_1^2 \frac{\sqrt{x}}{\log x} dx$ is divergent c) $\int_0^1 \frac{dx}{\sqrt{x}\sqrt[3]{1-x}}$ is convergent.

2. Examine the convergence of

a) $\int_2^\infty \frac{dx}{\sqrt{x^6+1}}$ b) $\int_0^\infty \frac{x^{2m}}{1+x^{2n}} dx$ c) $\int_0^\infty \frac{x^{p-1}}{1-x} dx$.

3. Show that the integral $\int_0^{\pi/2} log(sin\, x)\, dx$ is convergent and hence evalute it.

4. a) Using Cauchy's test show that $\int_0^\infty \frac{sin\, x}{x}\, dx$ is convergent.

b) Using Dirichlet's test show that $\int_0^\infty \frac{cos\, x}{\sqrt{x^2+x}} dx$ is convergent .

5. Discuss the convergence of $\int_0^1 log\sqrt{x}\, dx$ and hence evalute it.

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Practical No.- 8: Legendre Polynomials

1. Express $2 - 3x + 4x^2$ in terms of Legendre polynomials.
2. Express $x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre's polynomials.
3. Prove that a) $p_{2m+1}(0) = 0$ b) $p_{2m}(0) = (-1)^m \frac{(2m)!}{2^{2m}(m!)^2}$.
4. Prove that $\frac{1+z}{z\sqrt{(1-2xz+z^2}} - \frac{1}{z} = \sum_{n=0}^{\infty}(p_n + p_{n+1})\, z^n$.
5. Prove that $\int_0^\pi p_n(\cos\theta)\cos n\theta\, d\theta = \beta\left(n+\frac{1}{2},\frac{1}{2}\right)$.

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MTH-361: Vector Calculus

Practical No.-9: Vector Algebra

1. a) Determine the angles α, β *and* γ which the vector $\bar{r} = x\vec{i}+y\vec{j}+z\vec{k}$, makes with the positive directions of the co-ordinate axes and show that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

   b) Determine a set of equations for the straight line passing through the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$.

2. a) Determine a unit vector perpendicular to the plane of $\bar{A} = 2\vec{i} - 6\vec{j} - 3\vec{k}$ and $\bar{B} = 4\vec{i} + 3\vec{j} - \vec{k}$.

   b) Find an equation for the plane perpendicular to the vector $\bar{A} = 2\vec{i} + 3\vec{j} + 6\vec{k}$ and passing through the terminal point of the vector $= \vec{i} + 5\vec{j} + 3\vec{k}$. Also, Find the distance from the origin to the plane.

3. a) Find the angle which the vector $A = 3\vec{i} - 6\vec{j} + 2\vec{k}$ makes with the coordinate axes.

   b) If $\bar{A} = 2\vec{i} - 3\vec{j} - \vec{k}$ and $\bar{B} = \vec{i} + 4\vec{j} - 2\vec{k}$, find (i) $\bar{A}\times\bar{B}$ (ii) $\bar{B}\times\bar{A}$.

4. a) Find an equation for the plane determined by the points $P_1(2,-1,1)$, $P_2(3,2,-1)$ and $P_3(-1,3,2)$.

   b) Prove: (i) $\bar{A}\times\left(\bar{B}\times\bar{C}\right) = \bar{B}\left(\bar{A}\cdot\bar{C}\right) - \bar{C}\left(\bar{A}\cdot\bar{B}\right)$, (ii) $(\bar{A}\times\bar{B})\times\bar{C} = \bar{B}\left(A\cdot\bar{C}\right) - \bar{A}(\bar{B}\cdot\bar{C})$.

5. a) Find the unit tangent vector to any point on the curve $x = t^2+1,\ y = 4t-3,\ z = 2t^2-6t$ at point $t = 2$.

   b) If $A = 5t^2\vec{i} + t\,\vec{j} - t^3\vec{k}$ and $B = \sin t\,\vec{i} - \cos t\,\vec{j}$. Find (i) $\frac{d}{dt}(\bar{A}\cdot\bar{B})$ (ii) $\frac{d}{dt}(\bar{A}\times\bar{B})$ (iii) $\frac{d}{dt}(\bar{A}\cdot\bar{A})$.

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Practical No.-10: Vector Operator

1. a) Find $\nabla\phi$ *If* (i) $\phi = \ln|r|$, (ii) $\phi = \frac{1}{r}$ *and* (iii) Show that $\nabla^2\phi = 0$, where $r = x\vec{i}+y\vec{j}+z\vec{k}$.

   b) Find an equation for the tangent plane to the surface $2xz^2 - 3xy - 4x = 7$ at the point (1, -1, 2)

2. a) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction $2\vec{i} - \vec{j} - 2\vec{k}$.

   b) i) In what direction from the point $(2, 1, -1)$ is the directional derivative of $\phi = x^2yz^3$ a maximum ?

   ii) What is the magnitude of this maximum?

3. a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.

   b) Given $\phi = 2x^3y^2z^4$, show that $\nabla\cdot\nabla\phi = \nabla^2\phi$, where $\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

4. a) Prove (i) $\nabla\times\left(\overline{A} + \overline{B}\right) = \nabla\times\overline{A} + \nabla\times\overline{B}$, (ii) $\nabla\times\left(\phi\overline{A}\right) = (\nabla\phi)\times\overline{A} + \phi(\nabla\times\overline{A})$

   b) If $v = \omega\times r$, then prove that $\omega = \frac{1}{2}\text{curl } v$, where ω is constant vector.

5. Find constants a, b, c so that $\overline{V} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational and show that V can be expressed as the gradient of a scalar function.

6. If $\overline{A} = 2yz\vec{i} - x^2y\vec{j} + xz^2\vec{k}$, $\overline{B} = x^2\vec{i} + yz\vec{j} - xy\vec{k}$ and $\phi = 2x^2yz^3$, then find (i) $\left(\overline{A}\cdot\nabla\right)\phi$, (ii) $\overline{A}\cdot\nabla\phi$, (iii) $\left(\overline{B}\cdot\nabla\right)\overline{A}$, (iv) $\left(\overline{A}\times\nabla\right)\phi$, (v) $\overline{A}\times\nabla\phi$.

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Practical No.-11: Vector Integration

1. If $R(u) = (u - u^2)\vec{i} + 2u^3\vec{j} - 3\vec{k}$, find $(a) \int R(u)du$ and $(b) \int_1^2 R(u)\,du$.

2. If $\overline{A} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$, then evaluate $\int_C \overline{A}\cdot dr$ from $(0, 0, 0)$ to $(1, 1, 1)$ along the following paths C:

   a) $x = t,\ y = t^2,\ z = t^3$.

   b) The straight lines from $(0, 0, 0)$ to $(1, 0, 0)$, then to $(1, 1, 0)$, and then to $(1, 1, 1)$.

   c) The straight line joining $(0,0,0)$ and $(1, 1, 1)$.

3. Find the work done in moving a particle once around a circle C in the xy plane, if the circle has centre at the origin and radius 3 and if the force field is given by

$$\overline{F} = (2x - y + z)\vec{i} + (x + y - z^2)\vec{j} + (3x - 2y + 4z)\vec{k}.$$

4. Prove the following

   a) If $\overline{F}$ is a conservative field, prove that $curl\ \overline{F} = \nabla\times\overline{F} = 0$ (i.e. F is irrotational).

   b) Conversely, If $\nabla\times\overline{F} = 0$(i.e. $\overline{F}$ is irrotational), prove that $\overline{F}$ is conservative.

5. Evaluate $\iint_S \overline{A}\cdot n\, dS$, where $\overline{A} = 18z\vec{i} - 12\vec{j} + 3y\vec{k}$ and S is that part of the plane $2x + 3y + 6Z = 12$ which is located in the first octant.

6. Evaluate $\iint_S \phi\, n\, dS$, Where $\phi = \frac{3}{8}xyz$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ *and* $z = 5$.

7. Let $\overline{F} = 2xz\vec{i} - x\vec{j} + y^2\vec{k}$, Evaluate $\iiint_V \overline{F}\, dV$ Where V is the region bounded by the surfaces $x = 0,\ y = 0,\ y = 6,\ z = x^2,\ z = 4$.

8. Find the volume of the region common to the intersecting cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$.

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Practical No.-12: Integral Theorems

1. Evaluate $\int_{(0,0)}^{(2,1)} (10x^4 - 2xy^3)dx - 3x^2y^2dy$ along the path $x^4 - 6xy^3 = 4y^2$.

2. Evaluate $\oint_C (y - \sin x)dx + \cos x\, dy$, where C is the triangle with vertices

   O(0 , 0) , A(π /2 , 0), B(π /2 , 1) :

   (a) Directly, (b) By using Green's theorem in the plane.

3. Evaluate $\iint_S \overline{F} \cdot \mathrm{n} dS$, where $\overline{F} = 4xz\vec{\imath} - y^2\vec{\jmath} + yz\vec{k}$ and S is the surface of the cube bounded by $x = 0,\ x = 1,$ $y = 0,\ y = 1,\ z = 0,\ z = 1.$

4. Verify the divergence theorem for $\overline{A} = 4\mathrm{x}\vec{\mathrm{i}} - 2\mathrm{y}^2\vec{\mathrm{j}} + \mathrm{z}^2\vec{\mathrm{k}}$ taken over the region bounded by $x^2 + y^2 = 4,\ z = 0$ and $z = 3$.

5. Verify Stokes' theorem for $\overline{A} = (2x - y)\vec{\imath} - yz^2\vec{\jmath} - y^2z\vec{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

6. Prove $\oint dr \times \overline{B} = \iint_S \left(\overline{n} \times \nabla\right) \times B\, dS$.

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MTH-362: Real Analysis-II

Practical No.-13: Sequence of real numbers and Sequence of functions

1. Prove that a sequence of reals is Cauchy if and only if it is convergent .
2. Discuss the convergence of the sequence whose n^{th} term is $x_n = (1 + \frac{1}{n})^n$.
3. If $\{s_n\}_{n=1}^{\infty}$ is a Cauchy's Sequence of real numbers which has a subsequence converges to L, then show that $\{s_n\}_{n=1}^{\infty}$ itself converges to L.
4. Let $f_n(x) = \frac{x^n}{1+x^n}$, $0 \le x < 1$. Show that $\{f_n\}_{n=1}^{\infty}$ converges pointwise on [0, 1]. If $lim_{n\to\infty} f_n(x) = f(x)$, then does there exist $\mathrm{N} \in \mathrm{I}$ such that $|\mathrm{f_n}(x) - \mathrm{f}(x)| < \frac{1}{4}$ for all $\mathrm{n} \ge \mathrm{N}$, for all $x \in [0,1]$ simultaneously.
5. Let $f_n(x) = \frac{x}{1+nx}$ for $0 \le x \le 1$, then show that $\{f_n\}_{n=1}^{\infty}$ converges uniformly to zero.

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Practical No.-14: Series of real numbers

1. Discuss the convergence of the series

   a) $1 + x + x^2 + x^3 + \cdots$ b) $1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \cdots$ c) $(1 - 2) - \left(1 - 2^{\frac{1}{2}}\right) + \left(1 - 2^{\frac{1}{3}}\right) - \left(1 - 2^{\frac{1}{4}}\right) + \cdots$.

2. Examine the convergence of the series

   a) $\sum_{n=1}^{\infty} \frac{(n+1)}{10^{10}\,(n+2)}$ b) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)!}$ c) $\frac{1.2}{3.4.5} + \frac{2.3}{4.5.6} + \frac{3.4}{5.6.7} + \cdots$

3. Examine the convergence of the series

   a) $\sum_{n=1}^{\infty} \frac{5^n}{2^n+5}$ b) $\sum_{n=1}^{\infty} (\sqrt{n^2+1} - \sqrt{n^2-1}\,)$ c) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^3+1}$.

4. Discuss the convergence of the series

   a) $\sum_{n=1}^{\infty} \frac{n!}{5^n}$ b) $\sum_{n=1}^{\infty} \frac{x^n n!}{n^n}$.

5. Examine the convergence of

   a) $\sum_{n=1}^{\infty} \frac{(1+\frac{1}{n})^{2n}}{e^n}$ b) $\sum_{n=2}^{\infty} \frac{1}{n\,(logn)^p}$.

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Practical No.-15: Pointwise convergence and uniform convergence of series of functions

1. Show that the following series are uniformly convergent for all values of n:
   a) $\sum_{n=1}^{\infty} \frac{nx^2}{n^3+x^3}$
   b) $\sum_{n=1}^{\infty} \frac{sin(n^2+n^2x)}{n(n+2)}$.
2. Discuss the convergence of the series, $\sum_{n=0}^{\infty} x^n\, e^{-nx}$ on $[0, 10]$.
3. Show that
   a) $\sum_{n=1}^{\infty} \frac{1}{n^p+n^q x^2}$ is uniformly convergent for all values of n if $p > 1$.
   b) $\sum_{n=1}^{\infty} \frac{x}{n^p+n^q x^2}$ is uniformly convergent for $p + q > 2$.
4. Show that the series whose sum of First n^{th}term is $s_n(x) = \frac{n^2 x}{1+n^4 x^2}$ can be integrated term by term on [0, 1].
5. Prove that, if $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n}$ then $\int_0^1 f(x)dx = \sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.
6. Without finding the sum $f(x)$of the series

$$1 + \frac{x^2}{1!} + \frac{x^4}{2!} + \cdots + \frac{x^{2n}}{n!} + \cdots$$

   Show that $f'(x) = 2xf(x) \quad (-\infty < x < \infty)$.

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Practical No.-16: Fourier series

1. If the periodic function $f(x)$ is expressed in Fourier series expansion in $[-\pi,\ \pi]$ as

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

   Then determine the Fourier co-efficient $a_0,\ a_n$ and b_n.

2. Obtain the Fourier series for $f(x) = \begin{cases} 0 & \text{for } -\pi \le x \le 0 \\ x & \text{for } 0 \le x \le \pi \end{cases}$.

3. Obtain the Fourier series of the function $f(x) = e^{-x}$ in the interval $[0,\ 2\pi]$.

4. Obtain the Fourier series of the function $f(x) = x\ \sin x$ in $[-\pi,\ \pi]$.
   Hence deduce that $\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots$

5. Find the half range cosine series for $f(x) = x$ in $[0,\ \pi]$.

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Practical Course MTH-308
Based on MTH-353, MTH-354, MTH-363, MTH-364.
(With effect from June 2014)
Index

Practical No.	Title of Practical
1	Normal Subgroup and Isomorphism Theorems for groups
2	Permutation Groups
3	Quotient rings and Isomorphisms of rings
4	Polynomial rings
5	Kinematics
6	Rectilinear Motion
7	Uniplanar Motion
8	Central Orbit
9	Vector space, Subspace, Linearly Dependence and Independence
10	Basis and Dimensions ,
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12	Eigen values and Eigen vectors
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14	Linear differential equations of second order
15	Series solution of differential equations
16	Difference Equations

MTH-353: Abstract Algebra

Practical No.-1: Normal Subgroup and Isomorphism Theorems for groups

1. If H is subgroup of group G and $N(H) = \{g \in G : gHg^{-1} = H\}$, then prove that
   a) N(H) is normal subgroup of G
   b) H is normal subgroup of N(H).
2. If G is a group and H is a subgroup of index 2 in G. Prove that H is a normal subgroup of G.
3. a) Show that $< G, + >$ cannot be isomorphic to $< Q^*, \; . >$ where $Q^* = Q - \{0\}$ and Q = rational no.
   b) Show that any infinite cyclic group is isomorphic to $< Z, + >$, the group of integers.
4. If G = (Z,+) and N = (3Z, +) find the quotient group G/H.
5. a) Let Z = group of integer under addition then f : Z→ Z such that f(n)= $-\, n$. Show that f is homomorphism and hence automorphism.
   b) Let f : G→ G be a homomorphism, suppose f commutes with every inner automorphism of G, Show that K $= \{x \in G\colon f^2(x) = f(x)\}$ is a normal subgroup of G.

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Practical No.-2: Permutation Groups

1. Prepare a multiplication table of the permutations on S = {1, 2, 3} and show that S_3 is a group under the operation of permutation multiplication.
2. Express the following permutation ρ of degree 9 into the product of transpositions and find its order. Also find whether the permutation is odd or even, where $\rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 4 & 6 & 5 & 3 & 2 & 9 & 7 & 8 \end{pmatrix}$.
3. Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 4 & 1 & 8 & 2 & 5 & 7 \end{pmatrix}$ be a permutation in S_8.
   i) Find order of σ^{-1}
   ii) Express σ^{-1} as product of transpositions.
4. If σ = (1 3 5 4)(2 6 8)(9 7) and δ = (8 9 7 6)(5 4 1)(2 3), then find i) σ^{-1} and δ^{-1}
   ii) Order of σ , δ and σoδ.
5. Find $\sigma^{-1}\rho\sigma$ where ρ = (1 3 4)(5 6)(2 7 8 9) and $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 8 & 9 & 6 & 4 & 5 & 2 & 3 & 1 \end{pmatrix}$.

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Practical No.-3: Quotient rings and Isomorphisms of rings

1. a) Show that the intersection of two ideal is an ideal. But union may not be.
   b) Let a, b are commutative element of a ring R of characteristic 2. Show that $(a+b)^2 = a^2 + b^2 = (a-b)^2$.
2. a) If D is an integral domain and $na = 0$ for some $0 \neq a \in D$ and some integer $n \neq 0$, then show that the characteristic of D is finite.
   b) Let L be left ideal of ring R and let $\lambda(L) = \{ x \in R : \ xa = 0, \text{for all } x \in L\}$, then show that $\lambda(L)$ is an ideal of R.
3. a) Find all prime ideals and maximal ideals of $(Z_{12}, +_{12}, \times_{12})$.
   b) If R is division ring, then show that the centre Z(R) of R is a field.
4. Let $Z[i] = \{a + ib : a , b \in Z+\}$.
   a) Show that Z[i] is an integral domain.
   b) Find the field of quotients of Z[i].
5. Let R be a ring with identity 1 and $f : R \to R'$ be a ring homomorphism. Show that
   a) if R is an integral domain and $\ker(f) \neq 0$ then f(1) is an identity element of R′.
   b) if f is onto then f(1) is an identity element of R′.
6. Let $R^c = \{f : [0 , 1] \to R : f \text{ is continuous}\}$ be a ring under the operations $(f + g)(x) = f(x) + g(x)$ and $(fg)(x) = f(x)g(x)$ and $(R, +, \bullet)$ be a ring of real under usual addition and multiplication. Show that $\theta : R^c \to (R, +, \bullet)$ defined by $\theta(f) = f\left(\frac{1}{2}\right)$, for all $f \in R^c$, is onto ring homomorphism. Hence prove that $\{f \in R^c : f\left(\frac{1}{2}\right) = 0\}$is a maximal ideal of R^c.

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Practical No.-4: Polynomial rings

1. Let $f(x) = 2x^3 + 4x^2 + 3x - 2$ and $g(x) = 3x^4 + 2x + 4$ be polynomials over a ring $(Z_5, +_5, \times_5)$. Find
   a) $f(x) + g(x)$ b) deg $(f(x)\, g(x))$ c) zeros of $f(x)$ in Z_5.
2. Examine whether the polynomial $x^3 + 3x^2 + x - 4$ is irreducible over the field $(Z_7, +_7, \times_7)$.
3. Using Eisenstein's Criterion show that the following polynomials are irreducible over the field of rational numbers.
   a) $x^4 - 4x + 2$ b) $x^3 - 9x + 15$ c) $7x^4 - 2x^3 + 6x^2 - 10x + 18$.
4. Prove that the polynomial $1 + x + x^2 + x^3 + - - - + x^{p-1}$ is irreducible over the field of rational numbers, where p is a prime number.
5. Show that $\dfrac{Z_3[x]}{\langle x^2 + x + 1\rangle}$ is not an integral domain.
6. Show that $<x^2 + 1>$ is not a prime ideal of $Z_2[x]$.

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MTH-354: Dynamics

Practical No.-5: Kinematics

1. a) The co-ordinates of a moving point at time t are given by $x = a(2t+\sin 2t)$, $y = a(1-\cos 2t)$. Prove that its acceleration is constant.
   b) If the time of a body's descent in a straight line towards a given point vary directly as the square of the distance fallen through, prove that the acceleration is inversely proportional to the cube of the distance fallen through.
2. a) The velocities of a particle along and perpendicular to the radius from a fixed origin are λr^2 and $\mu\theta^2$, show that the equation to the path *is* $\frac{\lambda}{\theta} = \frac{\mu}{2r^2} + c$ and the components of acceleration are $2\lambda^2 r^2 - \frac{\mu^2\theta^4}{r}$ and $\lambda\mu r\theta^2 + 2\mu^2 \frac{\theta^3}{r}$
   b) Prove that the path of a point which possesses two constant velocities, one along a fixed direction and the other perpendicular to the radius vector drawn from a fixed point, is a conic section.
3. A particle describes an equiangular spiral $r = ae^{\theta}$ in such a manner that its acceleration has no radial component. Prove that its angular velocity is constant and that the magnitude of the velocity and acceleration is each proportional to r.
4. A point P describes, with a constant angular velocity of OP, an equiangular spiral of which O is the pole. Find its acceleration and show that its direction makes the same angle with the tangent at P as the radius vector OP makes with the tangent.
5. a) Prove that if the tangential and normal acceleration of a particle describing a plane curve be constant throughout the motion the angle ψ through which the direction of motion turns in time t is given by $\psi = A\log(1+Bt)$.
   b) A point moves in a curve so that its tangential and normal accelerations are equal and the tangent rotates with constant angular velocity. Show that the intrinsic equation of the path is of the form $s = Ae^{\psi} + B$.

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Practical No.-6: Rectilinear Motion

1. a) A body is projected vertically upwards with a velocity u, after time t another body is projected vertically upwards from the same point with a velocity v where $v < u$. If they meet as soon as possible, prove that $t = \frac{u-v+\sqrt{u^2-v^2}}{g}$
   b) A point moves with uniform acceleration and v_1, v_2, v_3 denote the average velocities in three successive intervals t_1, t_2, t_3. Prove that $\frac{v_1-v_2}{v_2-v_3} = \frac{t_1+t_2}{t_2+t_3}$.
2. For $1/m$ of the distance between two stations a train is uniformly accelerated and for $1/n$ of the distance it is uniformly retarded; it starts from rest at one station and comes to rest at the other. Prove that the ration of its greatest velocity to its average is $1+\frac{1}{m}+\frac{1}{n} : 1$.
3. a) A particle is performing a SHM of period T about a centre O and it passes through a point P (OP = b) with velocity v in the direction OP, prove that the time which elapses before its return to P is $\frac{T}{\pi}\tan^{-1}\frac{vT}{2\pi b}$.
   b) In a S. H. M. u, v, w be the velocities at distances a, b, c from a fixed point on the straight line which is not the centre of force, show that the period T is given by the equation

$$\frac{4\pi^2}{T^2}(b-c)(c-a)(a-b) = \begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}.$$

4. A particle of mass m executes S.H.M. in the line joining the points A and B on the smooth table and is connected with these points by elastic strings whose tensions in equilibrium are each T; show that the time of an oscillation is $2\pi\sqrt{\frac{m\,l\,l'}{T\,(l+l')}}$ where l, l' are the extensions of the strings beyond their natural lengths.
5. Two bodies M and M' are attached to the lower end of an elastic string whose upper end is fixed and are hung at rest, M' falls off ; show that the distance of M from the upper end of the string at time t is $a + b + c\cos\sqrt{\left(\frac{g}{b}\right)}\,t$ where a is unstretched length of the string, b and c the distances by which it would be extended when supporting M and M' respectively.

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Practical No.-7: Uniplanar Motion

1. a) If v_1, v_2 be the velocities at the ends of a focal chord of a projectile's path and u the horizontal component of the velocity show that $\frac{1}{v_1^2}+\frac{1}{v_2^2}=\frac{1}{u^2}$.

   b) Particles are projected from a point O in a vertical plane with velocity $\sqrt{2gk}$; prove that the locus of the vertices of their paths is the ellipse $x^2 + 4y(y - k) = 0$.

2) a) If t be the time in which a projectile reaches a point P in its path and t' the time from P till it reaches the horizontal plane through the point of projection, show that the height of P above a horizontal plane is $\frac{1}{2}gtt'$.

   b) If at any instant the velocity of the projectile be u and its direction of motion α to horizon, then it will be moving at right angles to this direction after the time $\frac{u}{g}cosec\propto$.

3) a) If the focus of a trajectories lies as much below the horizontal plane through the point of projection as the vertex is above, prove that the angle of projection is given by $sin\propto=\frac{1}{\sqrt{3}}$.

   b) A particle is projected so as to have a range R on the horizontal plane through the point of projection. If $\propto, \beta$ are the possible angles of projection and t_1, t_2 the corresponding times of flights, show that $\frac{t_1^2-t_2^2}{t_1^2+t_2^2}=\frac{sin\,(\propto-\beta)}{sin\,(\propto+\beta)}$.

4) Two particles are projected from the same point in the same vertical plane with equal velocities. If t, t' be the times taken to reach the other common point of their paths and T, T' the times to the highest point, show that $t\mathrm{T}+t'\mathrm{T}'$ is independent of the directions of projection.

5) A particle is projected under gravity with velocity $\sqrt{2ga}$ from a point at a height h above a level plane. Show that the angle of projection θ for the maximum range on the plane is given by $tan^2\theta=\frac{a}{a+h}$ and the maximum range is $2\sqrt{a(a+h)}$.

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Practical No.-8: Central Orbit

1. a) A particle moves in an ellipse under a force which is always directed towards the focus, find the law of force, the velocity at any point of its path and the periodic time.

   b) A particle describes the curve $r^n = a^n\, cosn\theta$ under a force F to the pole. Find the law of force.

2. a) If the central force varies as the distance from a fixed point, find the orbit.

   b) Prove that if the velocity at any point varies inversely as the distance of the point from the centre of force the orbit is an equiangular spiral.

3. a) A particle moving under a constant force from the centre is projected in a direction perpendicular to the radius vector with the velocity acquired in falling to the point of projection from the centre. Show that its path is $\left(\frac{a}{r}\right)^3=cos^2\frac{3}{2}\theta$.

   b) A particle acted on by a central attractive force $\frac{\mu}{r^3}$ is projected with a velocity $\frac{\sqrt{\mu}}{a}$ at an angle $\frac{\pi}{4}$ with its initial distance a, from the centre of force, prove that orbit is $r=ae^{-\theta}$.

4. A particle moves with a central acceleration $\frac{\mu}{r^5}$ and is projected from an apse at a distance a with a velocity equal to n times that which would be acquired from infinity, show that the other apsidal distance is $\frac{a}{\sqrt{n^2-1}}$.

5. Show that a particle can describe a rectangular hyperbola under a force from a fixed centre varying as the distance and show that the time the radius vector to the particle from the centre takes in sweeping out an angle θ from the vertex is given by $tan\theta=tanh(t\sqrt{\mu})$, μ being acceleration at unit distance. .

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MTH-363: Linear Algebra

Practical No.-9 : Vector space, Subspace, Linearly Dependence and Independence

1. Let V be the set of all ordered pairs (a, b) of real numbers with addition and multiplication defined on V by $(a, b) + (c, d) = (a+c, b+d)$ and $k(a, b) = (ka, 0)$. Show that V satisfies all axioms of a vector space except $1.u = u$.
2. If R^3(R) be the vector space of all ordered triads (a, b, c). Determine which of the following subsets of R^3(R) are subspaces :
   i) W = $\{(a, b, c): a, b, c \in$ R and $a+b+c = 0\}$. ii) W = $\{(a, b, c): a, b, c \in$ R and $a^2 + b^2 + c^2 \leq 1\}$.
3. Show that the set of all polynomials in one determinate x over a field F of degree less than or equal to n is a subspace of the vector space of all polynomials over a field F.
4. Determine whether the following vectors in R^4 are linearly dependent or independent .
   i) (1,2, –3,1) , (3,7,1, –2) , (1,3,7, –4)
   ii) (1,3,1, –2) , (2,5, –1,3) , (1,3,7, –2)
5. If a, b, c are linearly independent vectors in V(F), show that the vectors
   i) $a + b$, $b + c$, $c + a$ are linearly independent
   ii) $a + b$, $a - b$, $a - 2b + c$ are linearly independent
6. Which of the following set of polynomials of degree ≤ 3 over R are linearly independent :
   i) $x^3 - 3x^2 + 5x + 1$, $x^3 - x^2 + 8x + 2$, $2x^3 - 4x^2 + 9x + 5$.
   ii) $x^3 + 4x^2 - 2x + 3$, $x^3 + 6x^2 - x + 4$, $3x^3 + 8x^2 - 8x + 7$.
7. Show that the four vectors (1 , 1 , 1) , (1 , 2 , 3) , (1 , 5 , 8) , (1 , 1 , 1) generates R^3.
8. Find condition on a,b,c so that $(a, b, c) \in R^3$ belongs to the space generated by (2 , 1 , 0) , (1 , –1 , 2) , (0 , 3 , –4).

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Practical No.-10: Basis and Dimensions

1. Find the coordinate vectors $v_1 = (2, 7, -4)$ and $v_2 = (a, b, c)$ relative to the basis of S = $\{(1, 2, 0), (1, 3, 2), (0, 1, 3)\}$.
2. Find the basis and dimension of the solution space W of the following system of equations :
   $x + 2y + 2z - s + 3t = 0$, $x + 2y + 3z + s + t = 0$, $3x + 6y + 8z + s + 5t = 0$
3. Let W_1 and W_2 be two subspaces of R^4 given by
   $W_1 = \{(a, b, c, d) : b + c + d = 0\}$ $W_2 = \{(a, b, c, d) : a + b = 0, c = 2d\}$.
   Find the basis and dimension of, i) W_1 ii) W_2 iii) $W_1 \cap W_2$.
4. Determine whether or not (1 , 1 , 2) , (1 , 2 , 5) , (5 , 3 , 4) form a basis of R^3.
5. If W is the subspace of $R^4(R)$ spanned by the vectors (1 , –2 , 5 , –3), (2, 3 ,1, –4) , (3 , 8 , –3 , –5) then
   i) Find the basis and dimension of W. ii) Extend the basis of W to a basis of $R^4(R)$.
6. If V_1 and V_2 are the subspaces of a vector space $R^4(R)$ generated by the sets S_1={(1 , 1 , 0, –1) , (1 ,2 , 3,0) , (2 , 3, 3, –1)} and S_2={(1 , 2 , 2, –2) , (2 ,3, 2, –3) , (1 , 3, 4, –3)} respectively. Determine i) $\dim(V_1 + V_2)$ ii) $\dim(V_1 \cap V_2)$.
7. Find the rank and basis of the row space of the following matrix,
$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -10 & -5 \end{bmatrix}.$$
8. Extend (1, 1, 1, 1) and (2, 2, 3, 4) to a basis of $R^4(R)$.

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Practical No.-11: Linear Transformations

1. Let $f : R^4 \to R^3$ be the linear map defined by $f(x , y , z , t) = (x - y + z + t , 2x + 2y +3z–+4t , x -3y + 4z +5t)$. Find a basis and dimension of the image of f.
2. Show that the map T : R^2(R) → R^3(R) defined as $T(a , b) = (a + b , a - b , b)$ is a linear transformation. Find the range , rank, null space and nullity of T. Verify that, rankT + nullityT = dimR^3 (R).
3. Let T be a linear operator on R^3defined by $T(x_1, x_2, x_3) = (x_1 - x_2,\ x_2 - x_1,\ x_1 - x_3)$. Find the matrix of T to the basis B = {(1 , 0 , 1) , (0 , 1 , 1) , (1 , 1 , 0)}.
4. Let T be a linear operator on R^3defined by $T(x , y , z) = (2x, 4x - y, 2x+3y - z)$. Show that T is invertible and find the formulafor T^{-1}.
5. Show that the linear mapping T : $R^3 \to R^3$ defined by $T(x , y , z) = (x - 3y - 2z,\ y - 4z,\ z)$ is non-singular and find its inverse.
6. Find a linear transformation T :$R^3 \to R^2$ such that T(1 , 1 , 1) = (1 , 0) , T(1 , 1 , 0) = (2 , –1), T(1 , 0 , 0) = (4 , 3) .Also compute T(2,-3,5).
7. Let T be a linear operator on R^2 defined by $T(x , y) = (x +y , -2x + 4y)$. Compute the matrix of T relative to the basis {(1 , 1) , (1 , 2)}.
8. Let T : $R^2 \to R^3$ be a linear transformation defined by $T(x , y) = (y , - 5x–13y, - 7x+16y)$. Obtain the matrix of T in the following bases of $R^2$and R^3, where B_1 = {(3,1) , (5,2)}, B_2 = {(1 , 0, -1) . (-1 , 2 , 2) , (0 , 1 , 2)}.

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Practical No.-12: Eigen values and Eigen vectors

1. Find the Eigen values and corresponding Eigen vectors of the matrix A =$\begin{bmatrix} 3 & -4 \\ 2 & -6 \end{bmatrix}$.
2. Find the characteristics roots, their corresponding vectors and the basis for the vector space of a matrix A =$\begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$.
3. Verify Cayley-Hamilton Theorem for the matrix A =$\begin{bmatrix} 11 & -8 & 4 \\ -8 & -1 & -2 \\ 4 & -2 & -4 \end{bmatrix}$
4. Find the Eigen equation of the equation of the matrix A =$\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$ and verify it is satisfied by A. Hence find A^{-1}.
5. Show that the matrix A =$\begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$ is diagonalizable.
6. Find the matrix P, if it exists, which diagonalizes A, where A = $\begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$.
7. Find the minimum polynomial of the matrix A = $\begin{bmatrix} 2 & 2 & -5 \\ 3 & 7 & -15 \\ 1 & 2 & -4 \end{bmatrix}$.
8. Find the characteristics polynomial and the minimum polynomial of A =$\begin{bmatrix} 4 & -2 & 2 \\ 6 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$.

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MTH-364 : Differential Equations

Practical No.-13: Exact differential equations

1. a) Show that $x\frac{d^3y}{dx^3}+(x^2+x+3)\frac{d^2y}{dx^2}+(4x+2)\frac{dy}{dx}+2y=0$ is exact. Find its first integral.

   b) Show that $(2x^2+3x)\frac{d^2y}{dx^2}+(6x+3)\frac{dy}{dx}+2y=(x+1)e^x$ is exact. Hence solve it completely.

2. Find m if x^m is an integrating factor of $2x^2(x+1)\frac{d^2y}{dx^2}+x(7x+3)\frac{dy}{dx}-3y=x^2$, and hence solve it.

3. Solve

   a) $x^2y\frac{d^2y}{dx^2}+\left(x\frac{dy}{dx}-y\right)^2=0$

   b) $2x^2cosy.y''-2x^2siny.(y')^2+xcosy.y'-siny=logx$

4. Solve

   a) $y''=x^2sin^2x$

   b) $x^2\frac{d^4y}{dx^4}+1=0.$

5. Solve

   a) $\frac{d^2y}{dx^2}=sec^2x\ tany$

   b) $y^3y''=a.$

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Practical No.-14: Linear differential equations of second order

1. Solve by using the method of reduction of order

   a) $x^2y_2-2x(1+x)y_1+2(1+x)y=x^2$

   b) $xy''-(2x+1)y'+(x+1)y=x^3e^x$

2. Solve $(sinx-xcosx).y_2-xsinx.y_1+sinx.y=0,$ given that $y=sinx$ is a solution.

3. Solve $y''+(4cosec2x)y'+2tan^2x\ y=e^x\ cotx$ by changing the dependent variable.

4. Solve $y_2+2xy_1+(x^2+5)y=xe^{-x^2/2}$ by removal of the first derivative.

5. Solve by changing the independent variable.

   a) $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4cos\ [log(1+x)]$

   b) $y''+(tanx-3cosx)y'+2y\ cos^2x=cos^4x$.

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Practical No.-15: Series solution of differential equations

1. Discuss the singular points at x=0 and x= -1 for the following differential equation:
$$(x^2-1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0.$$
2. Discuss the singular points at x=0 and x= ∞ for the following differential equation:
$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-n^2)y=0.$$
3. Find the power series solution of the differential equation in powers of x about x=0:
   a) $(x^2+1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-xy=0$
   b) $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+x^2y=0$
4. Solve the following differential equations by the method of Frobenius:
   a) $9x(1-x)\frac{d^2y}{dx^2}-12\frac{dy}{dx}+4y=0$
   b) $x^2\frac{d^2y}{dx^2}+(x+x^2)\frac{dy}{dx}+(x-9)y=0$
   c) $x(1-x)\frac{d^2y}{dx^2}-3x\frac{dy}{dx}-y=0$
   d) $x\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0$

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Practical No.-16: Difference Equations

1. Form the difference equation corresponding to the following general solution:
   a) $y=ax^2+bx+c$
   b) $y=(a+bn)(-2)^n$
2. Solve the following difference equations:
   a) $y_{x+2}-y_{x+1}+y_x=0$ given that $y_0=1$ *and* $y_1=\frac{1+\sqrt{3}}{2}$
   b) $\Delta^3u_n-5\Delta u_n+4u_n=0$
3. Solve the following non-homogenous equations:
   a) $y_{x+2}-4y_{x+1}+4y_x=3^x+2^x+1$
   b) $y_{x+2}-4y_x=9x^2$
   c) $y_{n+2}-y_{n+1}+y_n=(n^2-n)2^n$
4. Formulate the Fibanocci difference equation and hence solve it.

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Practical Course MTH-309

Based on MTH-355(A/B), MTH-356(A/B), MTH-365(A/B), MTH-366(A/B).

(With effect from June 2014)

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MTH-355(A): Industrial Mathematics

Practical No.-1(A): Statistical Methods

1. An experiment consist of three independent tosses of a fair coin. Let X = The number of heads, Y = The number of heads runs. The length of head runs, a head run being defined as consecutive occurrence of at least two heads, its length then being the number of heads occurring together in three tosses of the coin. Find the probability function of (i) X (ii) Y (iii) Z (iv) X + Y (v) XY and construct probability tables and draw their probability charts.
2. a) The diameter of an electric cable, say X, is assumed to be a continuous random variable with p.d.f. :
   $f(x) = 6x(1-x)$, $0 \le x \le 1$. (i) check that f(x) is p.d.f. and
   (ii) determine a number b such that $P(x< b) = P(x > b)$

   b) A random variable X is distributed at random between the values 0 and 1 so that its p.d.f. is
   $f(x) = k x^2 (1-x^3)$, where k is constant. (i) Find the value of k.
   (ii) Using this value of k, find the mean and variance of the distribution.
3. a) A and B play a game in which their chances of winning are in the ratio 3:5. Find A's chance of winning at least three games out of the five games played.

   b) A department in a work has ten machines which may need adjustment from time to time. Three of these machines are old, each having a probability of (1/11) of needing adjustment during the day, and 7 are new having corresponding probability of (1/21). Assume that no machine needs adjustment twice on the same day. Using Binomial distribution determine the probability that on a particular day just two old and no new machine need adjustment.
4. a) A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day, is distributed as a Poisson distribution with mean 1.5. Calculate proportion of the days on which (i) neither car is used. (ii) the proportion of the days on which some demand is refused.

   b) In a Poisson frequency frequency distribution, a frequency corresponding to 3 successes is (2/3) times a frequency corresponding to 4 successes. Find the mean and standard deviation of the distribution.
5. Show that the exponential distribution "lacks memory" ie If X has an exponential distribution then for every constant $a \ge 0$ one has $P(Y \le x \mid X \ge a) = P(X \le x)$ for all x, where $Y = X - a$.

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Practical No.-2(A): Statistical Quality Control

1. Construct a control chart for mean and the range for the following data of 12 samples each of size 5. Examine whether the process is under control.

42	42	19	36	42	51	60	18	15	69	64	61
65	45	24	54	51	74	60	20	30	109	90	78
75	68	80	69	57	75	72	27	39	113	93	94
78	72	81	77	59	78	95	42	62	118	109	109
87	90	81	84	78	132	138	60	84	153	112	136

(For n = 5, are $A_2 = 0.577$, $D_3 = 0$ and $D_4 = 2.115$.)

2. The following data provides the values of sample mean $\bar{x}$ and the range R for ten samples of size 5 each. Calculate the values for central line and control limits for mean chart and range-chart , and determine whether the process is in control.

Sample No.	1	2	3	4	5	6	7	8	9	10
Mean ($\bar{x}$)	11.2	11.8	10.8	11.6	11.0	9.6	10.4	9.6	10.6	10.0
Range (R)	7	4	8	5	7	4	8	4	7	9

(For n = 5, are $A_2 = 0.577$, $D_3 = 0$ and $D_4 = 2.115$.)

3. The following figures give the number of defectives in 12 samples, each sample containing 200 items.

   22, 16, 18, 14, 38, 3, 20, 26, 26, 8, 0, 19

   Calculate the values for central line and the control limits for p-chart and comment if the process can be regarded in control or not?

4. The following data refers to visual defects found during inspection of the first ten samples of size 100 each. Calculate the control limits for the number of defective units (np-chart). Plot the control limits and the observations and state whether the process is under control or not.

Sample No.	1	2	3	4	5	6	7	8	9	10
Number of defectives	4	8	11	3	11	7	7	16	12	6

5. Twenty tape recorders were examined for quality control test. The number defects for each tape recorder is given below.

   2, 4, 3, 1, 1, 5, 3, 6, 7, 3, 1, 4, 2, 3, 1, 6, 4, 1, 1, 2.

   Prepare a c-chart. What conclusions do you draw from it.

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Practical No.-3(A): Queuing Theory

1. A repairman is to be hired to repair machines which break down at an average rate of 3 per hour. The break downs follow Poisson distribution. Non-productive time of machine is considered to cost Rs. 16 per hour. Two repairmen have been interviewed: one is slow but cheap, while the other is fast but expensive. The slow repairman charges Rs. 8 per hour and he services broken down machines at the rate 4 per hour. The fast repairman demands Rs. 10 per hour and he services at an average rate of 6 per hour. Which repairman should be hired? Assume 8-hour working day.
2. Workers come to tool store room to receive special tools (required by them) for accomplishing a particular project assigned to them. The average time between two arrivals is 60 seconds and the arrivals are assumed to be in Poisson distribution. The average service time (of the tool room attendant) is 40 seconds. Determine
   (a) average queue length,
   (b) average number of workers in the system including the worker being attended,
   (c) mean waiting time of an arrival,
   (d) the type of policy to established. In other words, determine whether to go in for an additional tool store room attendant which will minimize the combined cost of attendants' idle time and the cost of worker's waiting time. Assume the charges of a skilled worker Rs. 4 per hour and that of a tool store room attendant Rs. 0.75 per hour.
3. A branch of Punjab National Bank has only one typist. Since the typing works varies in length (number of pages to typed), the typing rate is randomly distributed approximating a Poisson distribution with mean service rate of 8 letters per hours. The letters arrive at a rate of 5 per hour during the entire 8-hour work day. If the typewriter is valued at Rs. 1.50 per hour, determine
   (a) equipment utilization.
   (b) the percent time that an arriving letter has to wait.
   (c) average system time.
   (d) average cost due to waiting on the part of the typewriter.
4. The milk plant at a city distributes its products by trucks, loaded at the loading dock. It has its own fleet of truck plus trucks of a private transport company. This transport company has complained that sometime its trucks have to wait in line and thus the company loses money paid for a truck and truck driver that is only waiting. The company has asked the milk plant management either to go in for a second loading dock or discount prices equivalence to the waiting time. The following data are available:

   Average arrival rate (all trucks)= 3 per hour.
   Average service rate = 4per hour

   The transport company has provided 40% of the total number of trucks. Assuming that these rates are random according to Poisson distribution, determine
   (a) The probability that the truck has to wait.
   (b) The waiting time of a truck that waits.
   (c) The expected waiting time of the company truck per day.
5. A person repairing radios finds that the time spent on the radio sets has an exponential distribution with mean 20 minutes. If the radios are repaired in the order in which they come in and their arrival is approximately Poisson with an average rate of 15 per 8-hour day,
   (a) What is the repairman's expected ideal time each day?
   (b) How many jobs are ahead of the average set just brought in?

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Practical No.-4(A): Sequencing and project scheduling by PERT and CPM

1. A machine operator has to perform two operations, turning and threading, on a number of different jobs. The time required to perform these operations (in minutes) for each job are known. Determine the order in which the job should be processed in order to minimize the total time required to turn out all the jobs. Also find the total elapsed time, and idle time for each machines.

Job	1	2	3	4	5	6	7
Time for turning (in minutes)	3	12	15	6	10	11	9
Time for threading (in minutes)	8	10	10	6	12	1	3

2. Four jobs 1, 2, 3,4 are to be processed on each of the five machines A,B,C,D,E in the order ABCDE. Find the total minimum elapsed time if no passing of jobs is permitted.

Jobs	Machine A	Machine B	Machine C	Machine D	Machine E
1	7	5	2	3	9
2	6	6	4	5	10
3	5	4	5	6	8
4	8	3	3	2	6

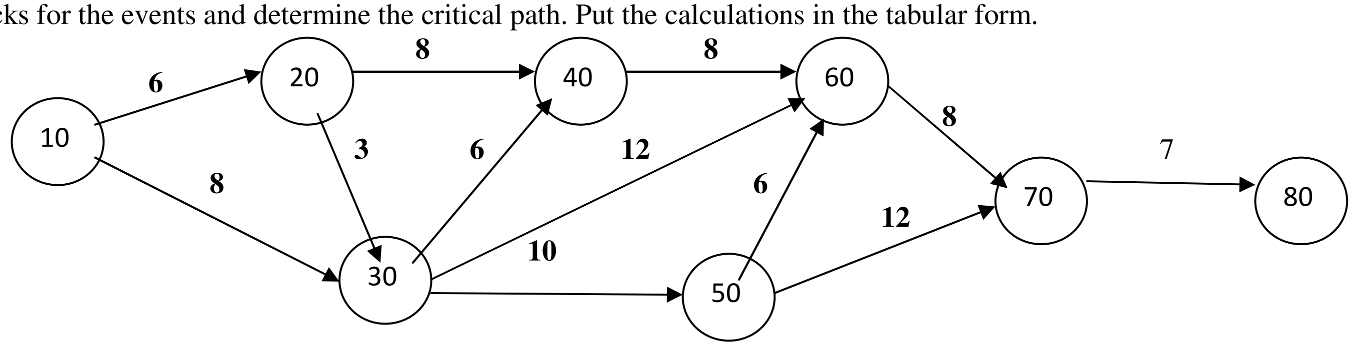
3. a) An assembly is to be made from two parts A and B both parts must be turned on a lathe and B must be polished where as, A need not be polished. The sequence of activities together with their predecessors are given. Draw a network diagram for the project and numbered it using Fulkerson's rule.

Activity	Predecessor activity
a: open work order	none
b: get material for A	a
c: get material for B	a
d: turn A on lathe	b
e: turn B on lathe	b, c
f : polish B	e
g: assemble a and B	d, f
h: pack	g

b) During a slack period, part of an assembly line is to be shut down for repair of a certain machine. While the machine is turned down the area will be painted. Construct a network for this machine rebuilding project based on the activity list furnished by the line for man as shown:

Activity	Activity Description	Restriction
A	Order new parts	A< B
B	Reassemble machine	B< E I
C	Tear out foundation	C< G
D	Dismantle machine	D< C,F,A
E	Paint area	E< A
F	Delivery parts to be repaired	F< H
G	Build new foundation	G< E
H	Pick up repaired parts	H< E
I	Clean up	____

4. Consider the network shown in the following figure. The activity time in day are given along the arrows. Calculate the slacks for the events and determine the critical path. Put the calculations in the tabular form.



MTH-355(B): Number Theory

Practical No.-1(B): Divisibility Theory

1. Show that the square of any odd integer is of the form $8k + 1$.
2. Given integers a, b, c and d verify the following
   a) If $a|b$ then $a|bc$ b) If $a|b,\ a|c$ then $a^2|bc$ c) $a|b$ iff $ac|bc$, where $c \neq 0$.
3. Use Mathematical Induction to establish
   a) $15 \mid 2^{4n} - 1$ b) $21 \mid 4^{(n+1)} + 5^{(2n-1)}$.
4. Use Euclidean Algorithm to obtain integers x and y satisfying
   a) $\gcd(306, 657) = 306x + 657y$. b) $\gcd(198, 288, 512) = 198x + 288y + 512z$.
5. Determine the solution in the integers of the following Diophantine equation
   a) $24x + 138y = 18$. b) $221x + 35y = 11$.

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Practical No.-2(B): Primes and their Distribution

1. a) Find all primes that divides 50!.
   b) If $p \geq q \geq 5, p$ and q are both primes then show that $24 \mid p^2 - q^2$.
2. Find the remainder when the sum $1! + 2! + 3! + \ . \ . \ . + 100!$ is divided by 12.
3. a) Show that the number 5233779 is divisible by 9.
   b) Show that the number 2587322568103 is divisible by 7 and 13.
4. a) Solve the following linear congruence's
   $i)\ 9x \equiv 21 \pmod{30}$ $ii)\ 140x \equiv 133\ (mod\ 301)$.
   b) Solve the linear congruence
   $i)\ x \equiv 3 \pmod{11}$, $ii)\ x \equiv 5 \pmod{19}$, $iii)\ x \equiv 10 \pmod{29}$.

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Practical No.-3(B): Congruence and Fermat's Theorem

1. Use Fermat's method to factor 119143.
2. Factorize $2^{11} - 1$ by Fermat's Factorization method.
3. a) Find the remainder when 72^{1001} is divided by 31.
   b) Find the remainder when 15! is divided by 17.
4. Verify that $5^{38} \equiv 4 \pmod{11}$.
5. Find the remainder when $2(28)!$ is divided by 31.

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Practical No.-4(B): Perfect numbers and Fibonacci numbers

1. If $2^k - 1$ is prime (k>1) then show that $2^{k-1}(2^k - 1)$ is a perfect number.
2. a) Show that the mersenne number M_{17} is a prime.
   b) Show that the mersenne number M_{23} is composite number.
3. a) Show that the Fermat number F_5 is divisible by 641.
   b) Show that there are infinitely many primes.
   c) Find all Pythagorean triples where terms are in A. P.
4. a) Represent each of the primes 109, 157, 197, 223 as a sum of two squares.
   b) Represent the integer 61, 92, 110 and 128 as sum of distinct Fibonacci numbers.

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MTH-356(A): Programming in C

Practical No.-5(A): Basic concept

1. Write a C program that will obtain the length and width of a rectangle from the user and compute its area and perimeter.
2. Write a C program to find the area of a triangle, given three sides.
3. Write a C program to find the simple interest, given principle, rate of interest and time.
4. Write a C program to read a five digit integer and print the sum of its digits.
5. Write a C program to convert a given number of days into months and days.
6. A computer manufacturing company has the following monthly compensation policy to their sales persons:

    Minimum base salary : 15000.00
    Bonus for every computer sold : 1000.00
    Commission on the total monthly sales : 2 per cent

    Assume that the sales price of each computer is fixed at the beginning of every month. Write a C program to compute a sales person's gross salary.

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Practical No.-6(A): Expressions and conditional statements

1. Write a C program that determines whether a given integer is odd or even and displays the number and description on the same line.
2. Write a C program that determines whether a given integer is divisible by 3 or not and displays the number and description on the same line.
3. Write a C program that determines the roots of the quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$.
4. Write a C program to print the largest of the three numbers using nested if . . .else statement.
5. Read four values a, b, c and d from the terminal and evaluates the ratio of $(a + b)$ to $(c - d)$and prints the result, if $c - d$ is not equal to zero.

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Practical No.-7(A): Looping

1. Write a C program to find the sum of even natural numbers from 1 to 100.
2. Write a C program that determines whether a given integer is prime or not.
3. Write a C program to prepare multiplication table from 11 to 30.
4. Write a C program to generate and print first n Fibonacci numbers.
5. Write a C program to find the following sum

    $$\text{Sum} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots\ldots + \frac{1}{n}.$$

6. Write a C program to get output by using for loop.

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1    2    3    4    5
1    2    3    4
1    2    3
1    2
1
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Practical No.-8(A): Arrays and Functions

1. Write a C program to sort N numbers in ascending order.
2. Write a C program to read two matrices and perform addition and subtraction of these matrices.
3. Write a C program to find the transpose of a given matrix.
4. Write a C program to read N integers (zero, positive, negative) into an array, and
   i) find the sum of negative numbers.
   ii) find the sum of positive numbers.
   iii) find the average of all input numbers.

   Output the various results computed with proper heading.
5. Given below is the list of marks obtained by a class of 50 students in an annual examination
   43, 65, 51, 27, 79, 11, 56, 61, 82, 09, 25, 36, 07, 49, 55, 63, 74, 81, 49, 37, 40, 49, 16, 75, 87, 91, 33, 24, 58, 78, 65, 56, 76, 67, 45, 54, 36, 63, 12, 21, 73, 49, 51, 19, 39, 49, 68, 93, 85, 59.

   Write a C program to count the number of students belonging to each of following groups of marks:
   0-9, 10-19, 20-29, …. , 100.

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MTH-356(B): Lattice Theory

Practical No.-5 (B): Posets

1. In any poset P, show that if $a, b, c \in$ P, $0 < a$ for number a and $a < b$, $b < c$ then $a < c$.
2. Prove that the two chains S = $\left\{0, ...,\frac{1}{n},...,\frac{1}{3},\frac{1}{2},1\right\}$ with respect to $\leq$ and T = $\left\{0, \frac{1}{2}, \frac{2}{3},....,\frac{n+1}{n},...,1\right\}$ with respect to $\leq$ are dually isomorphic.
3. Let A and B be two posets then show that A×B = $\{(a , b) : a \in$ A, $b \in$ B$\}$ is poset under relation defined by $(a_1 , b_1) \leq (a_2 , b_2)$ iff $a_1 \leq a_2$ in A , $b_1 \leq b_2$ in B.
4. Let S be set of even numbers up to 12. Define a relation $\leq$ on S as $a \leq b$ means a divides b. Draw poset diagram of S.
5. Let S be any set and L be a lattice. Let T = set of functions from S $\to$ L. Define relation '$\leq$' on T by f $\leq$ g iff f(x) $\leq$ g(x) for all $x \in$ T. Show that <T , $\leq$ > is a poset.

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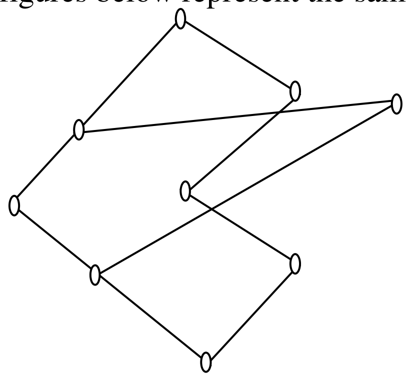
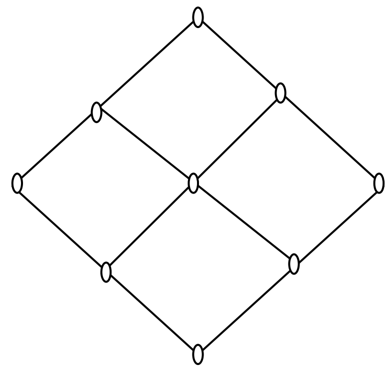
Practical No.-6(B): Lattices

1. Show that the figures below represent the same lattice:



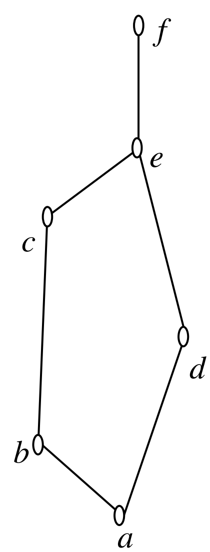


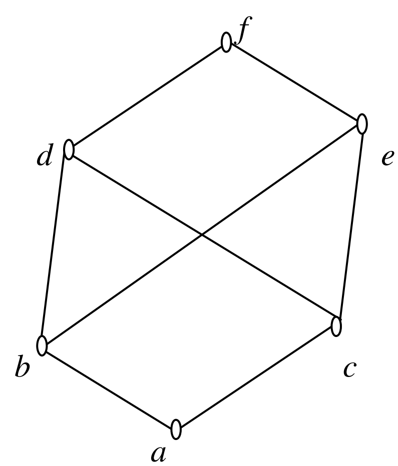
2. a) Show that a lattice L is a chain iff every non-empty subset of it is a sublattice.

   b) Let L be a lattice and $a, b \in$ L with $a \leq b$. Define $[a , b] = \{x \in \text{L} : a \leq x \leq b\}$. Show that $[a , b]$ is a sublattice of L.

3. Draw diagram of the lattice L of all 16 factors of natural number 216 where $a \leq b$ means a divides b.

4. Determine which of the following are lattices





5. Give an example of smallest modular lattice which is not distributive.

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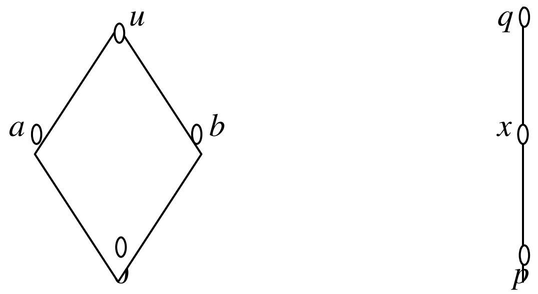
Practical No.-7(B): Ideals and Homomorphisms

1. a) Let N be the Lattice of all natural numbers under divisibility. Show that A = $\{1, p, p^2, ----\}$ where p is prime, forms an ideal of N.

   b) By an example show that union of two ideals need not be an ideal.

2. Let I be a prime ideal of lattice L. Show that L – I is dual prime ideal.
3. Let I and M be lattices given as



   a) Define $\sigma : L \to M$ such that $\sigma(0) = p$, $\sigma(a) = q$, $\sigma(b) = p$, $\sigma(u) = q$. Show that σ is a homomorphism.

   b) Define $\varphi : L \to M$ such that $\varphi(0) = p$, $\varphi(a) = x$, $\varphi(b) = x$, $\varphi(u) = q$. Show that φ is neither a meet nor a join homomorphism.

   c) Define $\psi : L \to M$ such that $\psi(0) = p$, $\psi(a) = p$, $\psi(b) = p$, $\psi(u) = q$. Show that ψ is a meet homomorphism but not a join homomorphism.

4. a) Show that no ideal of a complemented lattice which is proper sublattice can contain both an element and its complement.

   b) In a finite lattice prove that every ideal is principal ideal.

5. Show that in a complemented lattice the complement x' of an element x is unique.

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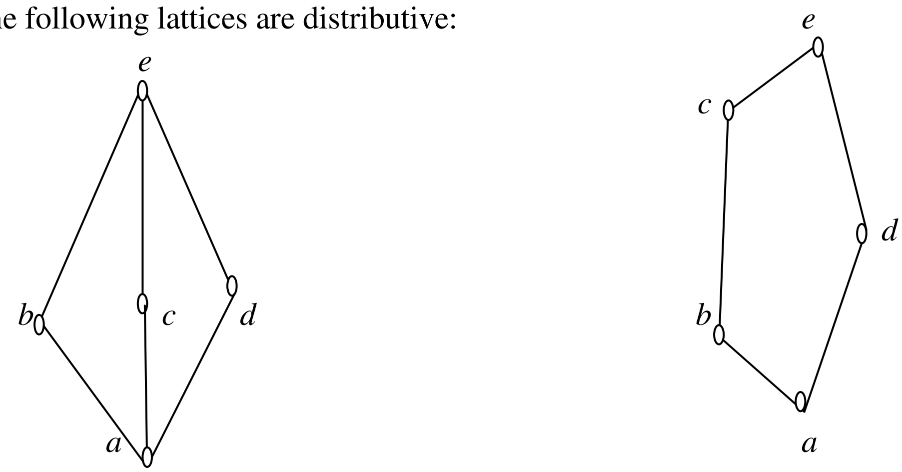
Practical No.-8(B): Modular and Distributive lattices

1. Determine which of the following lattices are distributive:



2. Let <R, +, •> be a ring and L be set of all ideals of R. Show that

   a) <L , ⊆ > form a lattice, where A, B ⊆ L, $A \wedge B = A \cap B$, $A \vee B = A \cap B = A + B$.

   b) <L , ⊆ > is a modular lattice.

3. a) Prove that homomorphic image of distributive lattice is distributive.

   b) Prove that homomorphic image of modular lattice is modular.

4. Show that lattice L is distributive if and only if $a, b, c \in L$, $a \wedge c \leq b \leq a \vee c$ □ $(a \wedge b) \vee (b \wedge c) = b = (a \vee b) \wedge (b \vee c)$.

5. If a, b, c are elements of modular lattice L with greatest element u and if $a \vee b = (a \wedge b) \vee c = u$, then show that $a \vee (b \wedge c) = b \vee (c \wedge a) = c \vee (a \wedge b) = u$.

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MTH-365(A): Operation Research

Practical No.-9(A): Linear programming problem

1. A farm is engaged in breeding pigs. The pigs are fed on various products grown on the farm. In view of the need to ensure certain nutrient constituents (call them X, Y, Z), it is necessary to buy two additional products, say A and B. One unit of product A contains 36 units of X, 3 units of Y and 20 units of Z. One unit of product B contains 6 units of X, 12 units of Y and 10 units of Z. The minimum requirement of X, Y and Z is 108 units, 36 units and 100 units respectively. Product A costs Rs. 20 per unit and product B Rs. 40 per unit. Formulate the above as a linear programming problem to minimize the total cost and solve the problem by using graphical method.
2. A firm manufacturers two products A and B on which the profits earned per unit are Rs. 3 and Rs. 4 respectively. Each product is processed on two machines M_1 and M_2.Product A requires one minute of processing time on M_1 and two minute on M_2 while B requires one minute on M_1 and one minute on M_2. Machine M_1 is available for not more than 7 hours and 30 minutes, while machine M_2 is available for 10 hours during any working day. Find the number of units of product A and B to be manufactured to get maximum profit.
3. Use simplex method to solve the following LPP:

   Maximize $z = 4x+10y$

   subject to constraints:

   $$2x+y \le 50$$
   $$2x+5y \le 100$$
   $$2x+3y \le 90$$
   $$x \ge 0 \quad \text{and} \quad y \ge 0.$$
4. Use penalty (or Big M) method to Maximize $z = 6x+4y$

   subject to constraints,

   $$2x + 3y \le 30$$
   $$3x +2y \le 24$$
   $$x + y \ge 3$$
   $$x \ge 0 \quad \text{and} \quad y \ge 0.$$

   Is the solution unique? If not, give two different solutions.
5. Use simplex method to solve the following LPP:

   Maximize $z = 3x+2y$

   subject to constraints:

   $$2x + y \le 2$$
   $$3x + 4y \ge 12$$
   $$x , y \ge 0.$$

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Practical No.-10(A): Transportation Problem

1. Determine the basic feasible solution of following transportation problem by
   (i) North-West Corner Method (ii)Matrix Minima Method (iii)Vogel's Approximation Method

Factory	Godowns						Stock available
	D_1	**D_2**	**D_3**	**D_1**	**D_5**	**D_6**	
O_1	6	4	8	4	9	6	4
O_2	6	7	13	6	8	12	5
O_3	3	9	4	5	9	13	3
O_4	10	7	11	6	11	10	9
Demand	4	4	5	3	2	3	

2. A company manufacturing air-coolers has two plants located at Mumbai and Calcutta with a weekly capacity of 200 units and 100 units, respectively. The company supplies air-coolers to its four showrooms situated at Ranchi, Delhi, Lucknow and Kanpur which has a demand of 75, 100, 100 and 30 units, respectively. The cost of transportation per unit (in Rs) is shown in the following table:

	Ranchi	Delhi	Lucknow	Kanpur
Mumbai	90	90	100	100
Calcutta	50	70	130	85

   Plan the production programme so as to minimize the total cost of transportation.

3. Consider the following transportation problem:

Factory	Godowns						Stock available
	1	**2**	**3**	**4**	**5**	**6**	
A	7	5	7	7	5	3	60
B	9	11	6	11	--	5	20
C	11	10	6	2	2	8	90
D	9	10	9	6	9	12	50
Demand	60	20	40	20	40	40	

   Is it possible to transport any quantity from factory B to Godown 5. Determine:
   (a) Initial solution by Vogel's Approximation Method.
   (b) Optimum Basic Feasible Solution.
   (c) Is the optimum solution unique? If not, find the alternative optimum basic feasible solution.

4. Find the optimum solution of the following transportation problem.

		Warehouses				
		P1	**P1**	**P1**	**P1**	Supply
Markets	**M1**	19	14	23	11	11
	M2	15	16	12	21	13
	M3	30	25	16	39	19
	Demands	6	10	12	15	

5. In the following transportation problem the cell entries are profits per unit in Rs.

		A	**B**	**C**	**D**	Capacity
	X	12	18	6	25	200
	Y	8	7	10	18	500
	Z	14	3	11	20	300
	Demands	180	320	100	400	

   Obtain the optimal solution of this transportation problem.

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Practical No.-11(A): Assignment Problem

1. A departmental head has four subordinates and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. His estimate of the time each man would take to perform each task is given in the matrix below:

	Men			
Tasks	**E**	**F**	**G**	**H**
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

How should the tasks be allocated, one to a man, so as to minimize the total man-hours?

2. A company has four zones A, B, C, D and four sales engineers P, Q, R, and S respectively for assignment. Since, the zones are not equally rich in sales potential, it is estimated that a particular engineer operating in a particular zone will bring the following sales:

Zone A: 4, 20,000 Zone B: 3, 36,000 Zone C: 2, 94,000 Zone D: 4, 62,000

The engineers are having different sales ability. Working under the same conditions their yearly sales are proportional to 14, 9, 11 and 8 respectively. The criteria of maximum expected total sales is to be met by assigning the best engineer to the richest zone, the next best to the second richest zone and so on. Find the optimum assignment and the maximum sales.

3. The following is the cost matrix of assigning 4 clerks to 4 key punching jobs. Find the optimal assignment if clerk 1 cannot be assigned to job 1:

Clerk	Job			
	1	**2**	**3**	**4**
1	--	5	2	0
2	4	7	5	6
3	5	8	4	3
4	3	6	6	2

What is the minimum total cost?

4. The owner of a small machine shop has four machinists available to assign to jobs for the

	A	**B**	**C**	**D**	**E**
1	62	78	50	101	82
2	71	84	61	73	59
3	87	92	111	71	81
4	48	64	87	77	80

Find the assignment of machinists to jobs that will result in a maximum profit. Which job should be declined?

5. Given the following matrix of set-up costs, show how to sequence production so as to minimize set-up cost per cycle:

From	To				
	A	**B**	**C**	**D**	**E**
A	∞	2	5	7	1
B	6	∞	3	8	2
C	8	7	∞	4	7
D	12	4	6	∞	5
E	1	3	2	8	∞

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Practical No.-12(A): Simulation

1. A bakery keeps stock of a popular brand of cake. Previous experience shows the daily demand pattern for the item with associated probabilities, as shown below:

Daily demand (number)	00	10	20	30	40	50
Probability	0.01	0.20	0.15	0.50	0.12	0.02

   Use the following sequence of random numbers to estimate the demand for next ten days.
   Random numbers: 48, 78, 19, 51, 56, 77, 15, 14, 68, 9.
   Also estimate the daily average demand for the cakes on the basis of simulated data.
2. Dr. strong is a dentist who schedules all her patients for 30 minutes appointments. Some of the patients take more or less than 30 minutes depending on the type of dental work to done. The following summary shows the various categories of work, their probabilities and the time actually needed to complete the work:

Category of service	Time required (minutes)	Probability of category
Filling	45 minutes	0.40
Crown	60 minutes	0.15
Cleaning	15 minutes	0.15
Extraction	45 minutes	0.10
Check-up	15 minutes	0.20

   Simulates the dentist's clinic for four hours and determine the average waiting time for the patients as well as idleness of the doctor. Assume that all the patients show up at the clinic at exactly their scheduled arrival time starting at 8 a.m. Use the following random numbers for handling the above problem:
   40, 82, 11, 34, 25, 66, 17, 79.
3. Production line turns out about 50truck per day; fluctuations occur for many reasons. The production can be described by a probability distribution as follows.

Production / day	45	46	47	48	49	50	51	52	53	54	55
Probability	0.03	0.05	0.07	0.10	0.15	0.20	0.15	0.10	0.07	0.05	0.03

   Finished trucks are transported by train at the end of the day. If the train capacity is only 5,what will be the average number of trucks waiting to be shipped and what will be the average number of empty spaces on the train? Use the following sequence of random numbers to simulate the production for next eight days:
   Random numbers are: 37, 35, 63, 25, 50, 71, 95, 16.
4. The Everalert Ltd. which has a satisfactory preventive maintenance system in its palnt, has installed a new Hot Air Generator based on electricity instead of fuel oil for drying its finished products. The Hot Air Generator requires periodic shutdown maintenance. If shutdown is scheduled yearly, the cost of maintenance will be as under:

Maintenance cost (Rs.)	Probability
1,50,000	0.30
2,00,000	0.40
2,50,000	0.30

   The cost are expected to be almost linear, that is, if the shutdown is scheduled twice a year the maintenance cost will be double. There is no previous experience regarding the time taken between breakdowns.

Cost associated with breakdowns will vary depending upon the periodicity of maintenance. The probability distribution of breakdown cost is estimated as shown:

Breakdown cost per annum(In Rs.)	Shutdown once a year	Shutdown twice a year
7,50,000	0.2	0.5
8,00,000	0.5	0.3
10,00,000	0.3	0.2

Simulate the total cost-maintenance and breakdown costs and recommend whether shutdown overhauling should be resorted to one a year or twice a year?
Use the random numbers 25, 44, 22, 32, 97 for one year maintenance cost.
And the random numbers 03, 50, 73, 87, 59 for one year breakdown cost.
Use the random numbers 42, 04, 82, 32, 91 for two year maintenance cost.
And the random numbers 54, 65, 49, 03, 56 for two year breakdown cost.

5. An Investment Company wants to study the investment projects based on three factors; market demand in units, profit per unit, and the investment required, which are independent of each other. In analyzing a new consumer product, the company estimates the following probability distributions for each of these three factors;

Annual demand		Profit per unit		Investment required	
Units	Probability	Rs.	Probability	Rs.	Probability
25,000	0.05	3.00	0.10	27,50,000	0.25
30,000	0.10	5.00	0.20	30,00,000	0.50
35,000	0.20	7.00	0.40	35,00,000	0.25
40,000	0.30	9.00	0.20		
45,000	0.20	10.00	0.10		
50,000	0.10				
55,000	0.05				

Using simulation process, repeat the trial 10 times, compute the return on investment for each trial taking these three factors into account. What is the most likely return? Use the following random numbers:
(30, 12, 16); (59, 09, 69); (63, 94, 26); (27, 08, 74); (64, 60, 61); (28, 28, 72); (31, 23, 57); (54, 85, 20); (64, 68, 18); (32, 31, 87).
In these bracket, the first random number is for annual demand, the second one is for profit and the last one is for the investment required.

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MTH-365(B): Combinatorics

Practical No.-9(B): Fundamental Principles of Counting.

1. Each user on a computer system has a password, which is six to eight characters long where each character is an upper case letter or a digit. Each password must contain at least one digit . How many possible password are there?
2. a) Use the product rule to show that the number of different subsets of a finite set S is $2^{|s|}$.
   b) Let A be a set with n elements .How many subsets does A have ?
3. A man, a woman, a boy, a girl, a dog and a cat are walking down a long and winding road one after the other.
   a) In how many ways can this happen?
   b) In how many ways can this happen if the dog immediately follows the boy?
   c) In how many ways can this happen if the dog comes first?
   d) In how many ways can this happen if the if the dog (and only the dog) is between the man and the boy?
4. a) How many binary sequences of r-bits long have even number of 1's ?
   b) What is the no. of diagonals that can be drawn in a polygon of n sides?
   c) Let n and r be non negative integers with $r \le n$. Then show that Σ=
   d) Show that if m and n are integers both greater than 2, then $R(m,n) \le R(m-1,n) + R(m, n-1)$
   e) Show that (a) $R(4,4) = 18$, b) $R(4,3) = 9$, c) $R(5,3) = 14$, d) $R(3,3) = 6$.

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Practical No.-10(B): The Principles of Inclusion–Exclusion.

1. Count the number of integral solutions to $x_1 + x_2 + x_3 = 20$ where $2 \le x_1 \le 5$; $4 \le x_2 \le 7$ and $-2 \le x_3 \le 9$.
2. How many positive integers not exceeding 1000 are divisible by 7 or 11?
3. Show that every sequence of $n^2 + 1$ distinct real number contains a subsequence of length n + 1 that is either strictly increasing or strictly decreasing.
4. Prove that given any 12 natural numbers, we can choose two of them such that their difference is divisible by 11.
5. Prove that the number of derangements of a set with n elements is $D_n = n![1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+...+(-1)^n\frac{1}{n!}]$.
6. Find the number of 4-digit positive integers the sum of the digits of which is 31.

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Practical No.-11(B): Generating Functions

1. a) Find the generating function of the sequence $a = 1, 1, 1, 1,…..$
   b) Find the generating function of the sequence $b = 1, 3, 9, …., 3^n,…$
   c) Find the exponential generating function for the sequence $t = {}^np_0, {}^np_1, {}^np_2, ..., {}^np_n$.
2. a) Find the number of positive integral solutions to the equation x + y + z = 10.
   b) Find the closed form for the generating function for each of the following sequences:
      (i) 0, 0, 1, 1, 1,……..
      (ii) 1, 1, 0, 1, 1, 1,……
      (iii) 1, 0, -1, 0, 1, 0, -1, 0, 1,……
      (iv) C(8,0), C(8,1), C(8,2),……, C(8,8), 0, 0, …..
      (v) 3, -3, 3, -3, 3, -3,……..
3. a) Find a formula to express $0^2 + 1^2 + 2^2 + …….. + n^2$ as a function of n.
   b) Find the coefficient of x^{20} in $(x^3 + x^4 + x^5 +……..)^5$.
4. a)Verify that for all $n \in Z^+$, $\binom{2n}{n} = \sum_{i=1}^{n}\binom{n}{i}^2$.

   b) Find a generating function for the sequence $A = \{a_r\}_{r=0}^{\infty}$

Where (i) $a_r = \begin{cases} 1 & \text{if } 0 \le r \le 2 \\ 3 & \text{if } 3 \le r \le 5 \\ 0 & \text{if } \quad r \ge 6 \end{cases}$ (ii) $a_r = \begin{cases} 1 & \text{if } 0 \le r \le 3 \\ 5 & \text{if } 4 \le r \le 7 \\ 0 & \text{if } \quad r \ge 8 \end{cases}$.

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Practical No.-12(B): Recurrence Relations

1. A person invests Rs. 10,000/- @ 12% interest compounded annually. How much will be there at the end of 15 years.
2. Solve the recurrence equation $a_n = a_{n-1} + 3$ with $a_1 = 2$ by
   a) Backtracking Method
   b) Forward Chaining Method
   c) Summation Method.
3. Solve the recurrence equation $t(n) = t(\sqrt{n}\ ) \ c \log_2 n$ with $t(1) = 1$.
4. Write down the first six terms of the sequence defined by $a_1 = 1$, $a_{k+1} = 3a_k + 1$ for $k \geq 1$. Guess a formula for a_n and prove that your formula is correct.
5. Find the solution of the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with initial conditions: $a_0 = 2$, $a_1 = 5$ and $a_2 = 15$.
6. Solve the recurrence equation, $a_r - 7\, a_{r-1} + 10\, a_{r-2} = 2^r$ with initial condition $a_0 = 0$ and $a_1 = 6$.
7. Solve the recurrence equation : , $a_r = \sqrt{a_{r-1} + \sqrt{a_{r-2} + \sqrt{a_{r-3} + \sqrt{\ldots\ldots\ldots\ldots}}}}$ with $a_0 = 4$.
8. If $f(x) = 1 + x + x^2 + \ldots\ldots + x^n + \ldots.$ and $g(x) = 1 - x + x^2 - x^3 + \ldots\ldots (-1)^n x^n + \ldots\ldots,$ find $f(x) + g(x)$ and $f(x).g(x)$.

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MTH-366(A): Applied Numerical Methods

Practical No.-13(A): Simultaneous Linear equations

1. Solve the following system by the method of triangularisation $2x - 3y + 10z = 3,\ \ -x + 4y + 2z = 20$ $5x + 2y + z = -12$.
2. Solve the following system by the method of factorization $x + 3y + 8z = 4,\ \ x + 4y + 3z = -2,$ $x + 3y + 4z = 1$.
3. Solve the following system by Crout's method $x + y + z = 3,\ \ 2x - y + 3z = 16,\ \ 3x + y - z = -3$.
4. Find the inverse of $\begin{bmatrix} 2 & -2 & 4 \\ 2 & 3 & 2 \\ -1 & 1 & -1 \end{bmatrix}$ by Crout's method.
5. Solve by Gauss-seidel method, the following system of equations $28x + 4y - z = 32,\ x + 3y + 10z = 24,$ $2x + 17y + 4z = 35$.
6. Solve by relaxation method, the equations $9x - y + 2z = 9,\ \ x + 10y - 2z = 15,\ \ 2x - 2y - 13z = -17$.

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Practical No.-14(A): Interpolation with Unequal Intervals

1. Given $\log_{10} 654 = 2.8156$, $\log_{10} 658 = 2.8182$, $\log_{10} 659 = 2.8189$, $\log_{10} 661 = 2.8202$. Find by using Newton's divided difference formula, the value of $\log_{10} 656$.
2. Use Lagrange's formula to find the form of $f(x)$, given

x	0	2	3	6
$f(x)$	648	704	729	792

3. Apply Lagrange's formula inversely to obtain the roots of $f(x) = 0$, given that
$$f(30) = -30 , f(34) = -13, f(38) = 3 \text{ and } f(42) = 18.$$
4. From the following data

x	1.8	2.0	2.2	2.4	2.6
y	2.9	3.6	4.4	5.5	6.7

Find x when $y = 5$, using iterative method.

5. Given $\cosh x = 1.285, find\ x$ by iterative method using the following data

x	0.736	0.737	0.738	0.739	0.740	0.741
$\cosh x$	1.28330	1.28410	1.28490	1.28572	1.23652	1.28733

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Practical No.-15(A): Numerical Differentiation and Integration

1. Find the first, second and third derivatives of $f(x)$ at $x = 1.5$ if

x	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	3.375	7.000	13.625	24.000	38.875	59.000

By using the derivatives of Newton's forward interpolation formula.

2. The population of a certain town (as obtained from census data) is shown in the following table.

Year	1951	1961	1971	1981	1991
Population (In thousand)	19.96	36.65	58.81	77.21	94.61

By using the derivatives of Newton's backward difference formula. Find the rate of growth of the population in the year 1981.

3. Evaluate $\int_0^{10} \frac{dx}{1+x^2}$ by using Trapezoidal rule.
4. The velocity v of a particle at distance s from a point on its path is given by the following table :

$s(ft)$	0	10	20	30	40	50	60
$v[ft/s]$	47	58	64	65	61	52	38

Estimate the time taken to travel 60 ft using Simpson's 1/3 rule.

5. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Romberg's method, correct to four decimal places. Hence find an approximate value of π.

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Practical No.-16(A): Numerical Solutions of Ordinary Differential Equations

1. Approximate y and z at $x = 0.1$ using Picard's method for the solution to the equations $\frac{dy}{dx} = z,\ \frac{dz}{dx} = x^3(y+z),$ given that $y(0) = 1$ and $z(0) = \frac{1}{2}$.
2. Given the differential equation $\frac{dy}{dx} = x - y^2$ with $y(0) = 1$, obtain the Taylor's series for $y(x)$ and find $y(0.1)$ upto four decimal places.
3. Using Euler's modified method, compute$y(0.2)$ for the differential equation $\frac{dy}{dx} = x + \left|\sqrt{y}\right|$ with y(0) = 1, take h = 0.2.
4. Using Runge-Kutta fourth order method find y(0.1), correct to four decimal places where $\frac{dy}{dx} = y - x$ with y(0) = 2.
5. Given $\frac{dy}{dx} = \frac{1}{x+y}$ $, y(0) = 2,\ y(0.2) = 2.0933, y(0.4) = 2.1755, y(0.6) = 2.2493,$ find $y(0.8)$ using Miline's method.
6. Using Adams-Bashforth method, find $y(1.4)$ given $y' = x^2(1+y),\ y(1) = 1,\ y(1.1) = 1.233,$ $y(1.2) = 1.548$ and $y(1.3) = 1.979$.

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MTH-366(B): Differential Geometry

Practical No.-13(B): Curves in Space

1. a) Find the equation of the tangent at the point u on the circular helix $x = a\cos u,\quad y = a\sin u,\quad z = cu$.
   b) Find the equation of the osculating plane at a general point on the curve given by $\bar{r} = (u, u^2, u^3)$.
2. a) Show that Serret - Frenete Formulae can be written in the form $\bar{t}^{I} = \bar{w} \times \bar{t}$, $\bar{n}^{I} = \bar{w} \times \bar{n}$ $\bar{b}^{I} = \bar{w} \times \bar{b}$ and determine $\bar{w}$.
   b) Prove that for any curve i) $\bar{t}^{I}.\ \bar{b}^{I} = -\kappa\tau$ ii) $\bar{t}^{I}.\ \bar{n}^{I} = 0$.
3. a) If a particle moving along a curve in space has velocity $\bar{v}$ and acceleration $\bar{f}$, then show that the radious of curvature ρ is given by $\frac{v^3}{|\bar{v} \times \bar{f}|}$.
   b) Prove that $[\ \bar{r}^{I}\ \bar{r}^{II}\ \bar{r}^{III}\ ] = \kappa^2\tau$.
4. Show that if the space curve $\bar{r} = \bar{r}\,(s)$ has constant torsion τ, then the curve $C_1, \bar{r}_1 = -\frac{-\bar{n}}{\tau} \int \bar{b}.ds$ has constant curvature $\pm\,\tau$.
5. a) Find radius of curvature of the helix $x = a\cos u, y = a\sin u,\quad z = au\tan\alpha$.
   b) Is the curve given by $x = a\cos u$, $y = a\sin u$, $z = cu$, a plane curve ? Justify.

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Practical No.-14(B): Curvatures

1. a) Show that the principal normal to a curve is normal to the locus of the centre of curveture at points where curvature κ is stationary.
   b) If the curve lies on the sphere then prove that $\rho + \frac{d^2\rho}{d\Psi^2} = 0$ where ψ is such that $d\psi = \tau\, ds$.
2. a) If R is the radius of the spherical curvature, then show that $R = \left| \frac{\bar{t} \times \bar{t}^{\mathrm{II}}}{\kappa^2\tau} \right|$
   b) Prove that the curve given by $x = a\cos u,\ y = 0,\ z = a\cos u$ lies on a sphere.
3. a) If the curve lies on the sphere then show that ρ and σ are related by $\frac{d}{ds}(\sigma\,\rho^1) + \frac{\rho}{\sigma} = 0$.
   b) Prove that $x^{\mathrm{III}\,2} + y^{\mathrm{III}\,2} + z^{\mathrm{III}\,2} = \frac{1}{\sigma^2\rho^2} + \frac{1+\rho^{\mathrm{I}2}}{\rho^4}$ where dashes denote differentiation with respect to 's'
4. a) Prove that the locus of the centre of curvature is an evolute only when the curve is plane.
   b) Prove that the involute of a circular helix are plane curve.
5. Show that the spherical indicatrix of the tangent of a curve is a circle if and only if the curve is a helix.

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Practical No.-15(B): Envelopes of Surfaces

1. a) Find the equation of the tangent plane and normal to the surface $2xz^2 - 3x^2y - 4x = 7$ at the point (1, -1, 2).
   c) Show that the sum of the squares of the intercepts on the coordinate axes made by the tangent plane to the surface $x^{2/3} + y^{2/3} + z^{2/3} = a^{2/3}$ is constant.
2. a) For paraboloid $r = (u, v, u^2 - v^2)$, find the metric.
   b) Calculate fundamental magnitudes for the surface of revolution ($u\cos v,\ u\sin v,\ f(u)$) show that parametric curves are orthogonal.
3. Find the envelope of the family of planes $3xt^2 - 2ty + z = t^3$ and show that its edge of regression is the curve of intersection of the surfaces $y^2 = zx$ and $xy = z$.
4. a) Find the envelope of the sphere $(x - a\cos\theta)^2 + (y - a\sin\theta)^2 + z^2 = b^2$.
   b) Find the envelope of $\ell x+my+nz=0$ where $a\ell^2 + bm^2 + cn^2 = 0$
5. A plane makes intercepts on the axes, so that $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{k^2}$ show that it envelopes a conicoid which has the axes as equal conjugate diameters.

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Practical No.-16(B): Developable Surfaces

1. a) Show that the line given by $y = t_x - t^3,\ z = t_y^3 - t^6$ generates a developable surface.
   b) Show that the line $x = 3t^2z + 2t\,(1-3t^4)\ y = -2tz + t^3\,(3+4t^2)$ generates a skew surface.
2. a) Find equations of the osculating developable surface of circular helix $x = a\cos u,\ y = a\sin u,\ z = cu$ for its edge of regression.
   b) Is surface $xy - z = 0$ is developable? Why?
3. Prove that the rectifying developable of a curve is the polar developable of its involutes and conversely.
4. a) Prove that the surface $z = c + \sqrt{xy}$ is developable.
   b) Prove that $xyz = a^2$ is not developable.
5. a) Prove that the curve itself is the edge of regression of the osculating developable.
   b) Prove that $z = y\sin x$ is a ruled surface.

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The equivalences for Old courses of T.Y.B.Sc. Mathematics is given as follows:

Semester	Old Course	New course (Equivalent course)
V	MTH : 311 – Metric Spaces	MTH : 351 – Metric Spaces
	MTH : 312 – Real Analysis – I	MTH : 352 – Real Analysis – I
	MTH : 313 – Modern Algebra	MTH : 353 – Abstract Algebra
	MTH : 314 – Dynamics	MTH : 354 – Dynamics
	MTH : 315(A) – Industrial Mathematics	MTH : 355(A) – Industrial Mathematics
	MTH : 315(B) – Number Theory	MTH : 355(B) – Number Theory
	MTH : 316(A) – Computer Organization & C Programming	MTH : 356(A) – Programming in C
	MTH : 316(B) – Lattice Theory	MTH : 356(B) – Lattice Theory
VI	MTH : 321 – Lebesgue Integration	MTH : 361 – Vector Calculus
	MTH : 322 – Real Analysis – II	MTH : 362 – Real Analysis – II
	MTH : 323 – Linear Algebra	MTH : 363 – Linear Algebra
	MTH : 324 –Differential Equations	MTH : 364 – Differential Equations
	MTH : 325(A) – Operation Research	MTH : 365(A) – Operation Research
	MTH : 325(B) – Combinatorics	MTH : 365(B) – Combinatorics
	MTH : 326(A) – Programming in C++	MTH : 366(A) – Applied Numerical Methods
	MTH : 326(B) – Differential Geometry	MTH : 366(B) – Differential Geometry
	MTH : 309 – Practical Course based on MTH: 315, MTH: 316, MTH: 325, MTH: 326	MTH : 309 – Practical Course based on MTH : 315, MTH: 316, MTH: 325, MTH: 326

Opportunities for mathematicians (Under Gradate)

Between one third and one half of all jobs requiring graduates are open to students of any discipline. Of course, mathematicians are eligible for these jobs. In addition, there are careers for which a degree in mathematics is either essential or a strong advantage. These fall into a number of general areas:

1. Scientific research, design and development
2. Management services and computing
3. Financial work
4. Statistical work
5. Teaching
6. Postgraduate study

Finally, a degree in mathematics does not train you for a specific job. Rather it gives you a range of skills which enable you to enter any of a wide range of careers. It is therefore a versatile qualification. By taking a mathematics degree, you are able to make your career choice when you are 21 rather than when you are 18. Your aspirations may well have changed during the intervening years. Moreover, you will have a clearer understanding of the work you would be doing and you will have been able to talk with representatives of the companies who will wish to employ you. Three years at a university/College will broaden your horizons in many ways. There is no need to narrow your career horizon while you are still at school unless you so wish.