

**NORTH MAHARASHTRA UNIVERSITY,
JALGAON**



FACULTY OF SCIENCE

**SYLLABUS FOR
T.Y. B. Sc.
(Mathematics)**

**To Be Implemented From
Academic Year 2017-18**

NORTH MAHARASHTRA UNIVERSITY JALGAON

Syllabus for T.Y.B.Sc (Mathematics)

With effect from June 2017 (Semester system)

The pattern of the examination of theory and practical papers is semester system. Each paper is of 100 marks(60 marks external and 40 marks internal).and practical course is of 100 marks(60 marks external and 40 marks internal).The examination will be conducted at the end of each semester.

STRUCTURE OF COURSES

Semester -I

MTH-351: Topics in Metric Spaces

MTH-352: Integral Calculus

MTH-353: Modern Algebra

MTH-354: Lattice Theory

MTH-355(A): C-Programming

OR

MTH-355(B): Elementary Number Theory

MTH-356(A): Vector Analysis

OR

MTH-356(B): Integral Transform

MTH-357: Prac. Course based on (MTH-351,352)

MTH-358: Prac. Course based on (MTH-353,354)

MTH-359: Prac. Course based on (MTH-355,356)

Semester-II

MTH-361: Measure and integration

Theory

MTH-362: Method of Real Analysis

MTH-363: Linear Algebra

MTH-364: Ordinary and Partial Differential
Equation

MTH-365(A): Optimization Techniques

OR

MTH-365(B): Dynamics

MTH-366(A): Applied Numerical Methods

OR

MTH-366(B): Differential Geometry

MTH-367: Prac. Course based on (MTH-361,362)

MTH-368: Prac. Course based on (MTH-363,364)

MTH-369: Prac. Course based on (MTH-365,366)

N.B.: Work load should not be increased by choosing optional courses.

SEMESTER –I

MTH – 351 : Topics in Metric Spaces.

- Unit – 1 : Metric Spaces.** **Periods - 15 , Marks-15**
- 1.1 Equivalence and Countability.
 - 1.2 Metric Spaces
 - 1.3 Limits in Metric Spaces
- Unit -2 : Continuous functions on Metric Spaces.** **Periods -15 , Marks -15**
- 2.1 Reformulation of definition of continuity in Metric Spaces.
 - 2.2 Continuous function on Metric Spaces.
 - 2.3 Open Sets
 - 2.4 Closed Sets
 - 2.4 Homeomorphisms.
- Unit -3 Connectedness and Completeness of Metric Spaces** **Periods -15 , Marks 15**
- 3.1 More about Sets
 - 3.2 Connected Set
 - 3.3 Bounded and Totally bounded sets
 - 3.4 Complete Metric Spaces
 - 3.5 Contraction Mapping on Metric Spaces.
- Unit -4 Compactness of Metric Spaces** **Periods -15 , Marks 15**
- 4.1 Compact Metric Spaces .
 - 4.2 Continuous function on compact Metric Spaces .
 - 4.3 Continuity of inverse function
 - 4.4 Uniform Continuity

Recommended Books : [1] R.R. Goldberg, Methods of Real Analysis, John Weley and Sons, Inc. New York, 2nd Edition, 1976

Chapter I : 1.5 ,1.6 Chapter IV : 4.2, 4.3

Chapter V : 5.2,5.3, 5.4, 5.5 Chapter VI : 6.1,6.2,6.3.6.4,6.5,6.6,6.7,6.8

Reference Book :

[1] D.Somsundaram and B. Chaudhari, A first Course in Mathematical Analysis(1996) Narosa Publishing House.

[2] S.C.Malik & Savita Arora, Mathematical Analysis (2010).

MTH-352 : Integral Calculus

Unit 1: Riemann Integration

Periods-15, Marks-15

- 1.1 Definition and existence of the integral. The meaning of $\int_a^b f(x)dx$ when $a \leq b$.
Inequalities for integrals.
- 1.2 Refinement of partitions.
- 1.3 Darboux theorem (without proof)
- 1.4 Condition of integrability.
- 1.5 Integrability of the sum and difference of integrable functions.
- 1.6 The integral as a limit of a sum (Riemann sums) and the limit of a sum as the integral and its applications.
- 1.7 Some integrable functions.

Unit :2 Mean Value Theorem of integral calculus

Periods-15, Marks-15

- 2.1 Integration and Differentiation (the primitive)
- 2.2 Fundamental theorems of integral calculus.
- 2.3 The first Mean Value Theorem.
- 2.4 The generalized first Mean Value Theorem.
- 2.5 Abel's lemma (without proof)
- 2.6 Second Mean Value Theorem. Bonnets form and Karl Weierstrass form.

Unit 3: Improper Integrals

Periods-15, Marks15

- 3.1 Integration of unbounded functions with finite limits of integration.
- 3.2 Comparison test for convergence at a of $\int_a^b f(x)dx$.
- 3.3 Convergence of improper integral $\int_a^b \frac{dx}{(x-a)^n}$
- 3.4 Cauchy's general test for convergence at a point a of $\int_a^b f(x)dx$.
- 3.5 Absolute convergence of improper integral $\int_a^b f(x)dx$.
- 3.6 Convergence of integral with infinite range of integration.
- 3.7 Comparison test for convergence at ∞ .
- 3.8 Cauchy's general test for convergence at ∞ .
- 3.9 Convergence of $\int_a^\infty \frac{dx}{x^n}$, $a > 0$.
- 3.10 Absolute convergence of improper integrals with infinite range of integration.
- 3.11 Abel's test and Dirichlet's test for convergence of $\int_a^\infty f(x)dx$. (Statements and examples only).

Unit 4: Legendre Polynomials

Periods-15, Marks-15

- 4.1 Legendre's equation and its solution.
- 4.2 Legendre's function of the first kind.
- 4.3 Generating function for Legendre's polynomials.
- 4.4 Trigonometric series for $p_n(x)$.
- 4.5 Laplace's definite integrals for $p_n(x)$ (first and second integrals).

Recommended Books :

- 1) S.C.Malik & Savita Arora, Mathematical Analysis (2010).
- 2) M.D. Raisinghania, Ordinary and Partial Differential Equations, S Chand Publication 8th Edition (2005)

Reference Books :

- 1) R.R. Goldberg, Methods of Real Analysis, John Weley and Sons, Inc. New York, 2nd Edition, 1976
- 2) S. K. Chatterjee, Mathematical Analysis

MTH-353: Modern Algebra

Unit 1 : Normal Subgroup and Isomorphism Theorems for groups

Periods – 15, Marks – 15

- 1.1 Normal subgroups
- 1.2 Quotient groups, Isomorphism theorems for groups
- 1.3 Isomorphism theorems for groups and examples
- 1.4 Generator of a subgroup
- 1.5 Commutator subgroup
- 1.6 Automorphism and inner automorphism

Unit 2 : Permutations

Periods – 15, Marks – 15

- 2.1 Permutations
- 2.2 Cycles of permutation
- 2.3 Disjoint permutations
- 2.4 Permutation groups

Unit 3 : Quotient rings and Isomorphisms of rings

Periods – 15, Marks – 15

- 3.1 Revision of Ring, integral domain, field, zero divisors, and basic properties
- 3.2 Characteristics of a ring
- 3.3 Subrings, ideals, left ideals, right ideals, principal ideals, prime ideals, maximal ideals.
- 3.4 Quotient rings
- 3.5 Field of quotients of an integral domain (Definition & Examples only)
- 3.6 Homomorphism of rings, Isomorphism theorems for rings.

Unit 4 : Polynomial Rings

Periods – 15, Marks – 15

- 4.1 Definition of a polynomial ring, Properties of polynomial rings
- 4.2 Division Algorithm
- 4.3 Reducible and Irreducible polynomials
- 4.4 Eisenstein's Criterion.

Recommended Book :

- 1) V. K. Khanna and S. K. Bhambri, A course in Abstract Algebra (2013, 3rd Edition), Vikas Publishing House Pvt. Ltd. New Delhi.

2) J. B. Fraleigh, A first course in Abstract Algebra, (1967, 3rd Edition).

Reference Books: 1) N. S. Gopalkrishnan, University Algebra.

MTH -354: Lattice Theory**Unit 1: Posets****Periods -15 , Marks -15**

- 1.1 Posets and Chains
- 1.2 Diagrammatical Representation of posets
- 1.3 Maximal and Minimal elements of subset of a poset, Zorn's Lemma (Statement only)
- 1.4 Supremum and infimum
- 1.5 Poset isomorphism
- 1.6 Duality Principle.

Unit 2:Lattices**Periods -15 , Marks -15**

- 2.1 Two definitions of lattice and equivalence of two definitions
- 2.2 Modular and Distributive inequalities in a lattice .
- 2.3 Sublattice and Semilattice
- 2.4 Complete lattice

Unit 3: Ideals and Homomorphisms.**Periods -15, Marks -15**

- 3.1 Ideals ,Union and intersection of Ideals
- 3.2 Prime Ideals
- 3.3 Principal Ideals
- 3.4 Dual Ideals
- 3.5 Principal dual Ideals
- 3.6 Complements , Relative Complements
- 3.7 Homomorphisms , Join and meet homomorphism

Unit 4: Modular and Distributive Lattices**Periods 15, Marks -15**

- 4.1 Modular lattice
- 4.2 Distributive lattice
- 4.3 Sublattice of Modular lattice
- 4.4 Homomorphic image of Modular lattice
- 4.5 Complemented and Relatively complemented lattice

Recommended Book:

- 1) Vijay K.Khanna, Lattices and Boolean Algebra (Chapter -2,3,4), (2004, 2nd Edition), Vikas Publ. Pvt. Ltd
- 2) George Gratzer, General Lattice Theory, (2013, 2nd Editon), Birkhauser

MTH-355(A)

C-Programming

Unit 1: Basic concepts:**Periods – 13 , Marks - 15**

- 1.1 Introduction
- 1.2 character set
- 1.3 C tokens, keywords
- 1.4 Constants
- 1.5 variables,data types
- 1.6 variables, symbolic constants
- 1.7 over flow, under flow
- 1.8 operators of arithmetic, relational, logical, assignment , increment and decrement, conditional and special type.

Unit II: Expressions and conditional statements:**Periods – 13 , Marks – 15**

- 2.1 Arithmetic expression and its evaluation precedence of arithmetic operators type
- 2.2 Conversion, operator precedence, mathematical functions
- 2.3 Reading and writing a character
- 2.4 Formatted input and out put
- 2.5 Decision making , if, is-else, else-if, switch and goto statements.

Unit III: Loops: Decision making and Looping:**Periods – 13 , Marks – 15**

- 3.1 Sentinel loops. While loop, do-while loop and for statements.
- 3.2 Jump in loops, continue, break and exit statements.

Unit IV: Arrays and Functions:**Periods – 13 , Marks – 15**

- 4.1 One dimensional, two dimensional and multidimensional arrays. Declaration and initialization of arrays.
- 4.2 Need for user defined functions, multi-function program
- 4.3 Elements of function,definition of functions, return values and their types

Recommended Book:

1. E. Balagurusamy, Programming in ANSI C ,(2012) Mcgraw-Hill company.

Reference Book :

1. Yashwant Kanitkar, LET Us C 14TH Edition 2016 B.P.B. Publication

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MTH-355(B): Elementary Number Theory

Unit 1 : Prime numbers and Diophantine Equations Periods – 15, Marks – 15

- 1.1 The Fundamental Theorem of Arithmetic
- 1.1 The Sieve of Eratosthenes
- 1.3 The Goldbach Conjecture
- 1.4 The Diophantine Equation $ax + by = c$

Unit 2 : The theory of congruences Periods – 15, Marks – 15

- 2.1 Basic Properties of Congruence
- 2.2 Binary and decimal representations of integers.
- 2.4 Linear Congruences and the Chinese Remainder Theorem.
- 2.5 Fermat's Factorization Method

Unit 3 : Fermats Theorem and Perfect numbers Periods – 15, Marks – 15

- 3.1 The Little Theorem and pseudoprimes
- 3.2 Wilson's Theorem
- 3.3 Perfect Numbers
- 3.4 Mersenne Numbers
- 3.5 Fermat's Numbers

Unit 4 : Fibonacci numbers and Finite continued fractions Periods – 15, Marks – 15

- 4.1 The Fibonacci Sequence
- 4.2 Certain Identities Involving Fibonacci Numbers.
- 4.3 Finite continued fractions

Recommended Book :

- 1) David M. Burton, Elementary Number Theory (Sixth Edition) -. (19780) (Tata McGraw-Hill Edition, New Delhi)
- Ch.3 : 3.1 to 3.3, Ch . 2 : 2.5, Ch.4 : 4.2 to 4.4, Ch.5 : 5.2 to 5.4, Ch.11 : 11.2 to 11.4, Ch 14 : 14.2 to 14.3, Ch 15: 15.2

Reference Books :

- 1) T. M. Apostol Introduction to Analytic Number Theory – (1972)(Springer International student Edition)
- 2) Hari Kishan , Number Theory –, (2014)Krishna Prakashan Media (p) Ltd, Meerat.

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MTH-356(A) Vector Calculus

Unit - 1 : Vector Functions Periods - 13, Marks – 15

- 1.1 Vector functions of a single variable.
- 1.2 Limits and continuity.
- 1.3 Differentiability, Algebra of differentiation.
- 1.4 Curves in space, Velocity and acceleration.
- 1.5 Vector function of two or three variables.
- 1.6 Limits, Continuity, Partial Differentiation.

Unit - 2 : Differential Operators : Periods - 13, Marks – 15

- 2.1 Gradient of a scalar point function , Properties of a gradient
- 2.2 Geometric meaning of a gradient.
- 2.3 Directional derivative of a scalar point function.
- 2.4 Divergence and Curl of vector point function and their properties.

Unit – 3 : Vector Integration: Periods - 13, Marks – 15

- 3.1 Ordinary integrals of vectors.
- 3.2 Line Integration.
- 3.3 Surface Integrals.
- 3.4 Volume Integrals.

Unit – 4 : Integral Theorems Periods - 13, Marks – 15

- 4.1 Green's Theorem in the plane.
- 4.2 Stokes Theorem.
- 4.3 Gauss Divergence Theorem.

Recommended Text Books:

1. Murray R Spiegel, Vector Analysis By, (1959) Schaum Series, Mcgraw Hill Book Company.
 2. Shanti Narayan And P.K. Mittal Vector Calculus ,4th edition (1987) S.Chand & Co., New Delhi.
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MTH- 356 (B) Integral transforms

1. Fourier transforms:-

Periods - 13, Marks – 15

Complex and exponential form of Fourier series, Fourier integral, equivalent form of Fourier integral, sine and cosine integrals, Fourier transform, Fourier cosine transform, Fourier sine transform, useful results for evaluating the integrals in Fourier transforms, inverse Fourier transforms.

2. PROPERTIES AND THEOREMS OF FOURIER TRANSFORMS:-

Periods - 13, Marks – 15

Linearity property, change of scale property, shifting property, Modulation theorem, convolution theorem, finite Fourier transforms, finite Fourier cosine transform, finite Fourier sine transform. Fourier transform of the derivatives of a function, Applications of Fourier transforms to boundary value problems.

3. Z-TRANSFORM :-

Periods - 13, Marks – 15

Basic preliminary, Z-transforms, inverse Z-transform, Z-transform pair, uniqueness of inverse Z-transform, Properties of Z-transforms, Z-transform of some standard sequences, illustrations on Z-transforms.

4. INVERSE Z- TRANSFORM:-

Periods - 13, Marks – 15

Power series method, partial fraction method, inversion integral method, solution of difference equations with constant coefficients using Z-transform, relationship of Z-transform with Fourier transform.

Reference Book:- 1. Davies, Brian, 3rd edition Integral Transforms and Their Applications (2002) Springer Verlag, New York

2. Lokenath Debnath, Dambaru Bhatta, 2nd edition 2014, Integral Transforms and Their Applications, Second Edition

- 2006

SEMESTER –I**T.Y.B.Sc Mathematical Practical Course MTH-357****Based on MTH-351, MTH-352**

Index

Practical No.	Title of Practical
1	Metric Spaces
2	Continuous functions on Metric Spaces.
3	Connectedness and Completeness of Metric Spaces
4	Compactness of Metric Spaces
5	Riemann Integration
6	Mean Value Theorem of integral calculus
7	Improper Integrals
8	Legendre Polynomials

MTH-351 : Metric Spaces

Practical No. 1 - Metric spaces

1. If A_1, A_2, \dots, A_n are countable sets then show that $\bigcup_{n=1}^{\infty} A_n$ is countable.
2. Show that the intervals $(0,1)$ and $[0,1]$ are equivalent.
3. Show that the intervals $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $(-\infty, \infty)$ are equivalent
4. Let $x = (x_1, x_2)$ $y = (y_1, y_2)$ be any two points in \mathbb{R}^2 . Define $\rho : \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R}$ by $\rho(x,y) = \max(|x_1 - y_1|, |x_2 - y_2|)$ show that ρ is metric for \mathbb{R}^2 .
5. Let $d : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ be defined by $d(x,y) = \frac{|x-y|}{1+|x-y|} \quad \forall x, y \in \mathbb{R} = \text{set of reals}$ show that d is a metric for \mathbb{R} .
6. If $\{X_n\}_{n=1}^{\infty}$ is a convergent sequence in a discrete metric space R_d then show that there exists a positive integer N such that $X_n = X_{n+1} = X_{n+2} = \dots$
7. If $\{X_n\}_{n=1}^{\infty}$ is a Cauchy sequence of points in the metric space M and if $\{X_n\}_{n=1}^{\infty}$ has a subsequence which converges to $x \in M$, then prove that $\{X_n\}_{n=1}^{\infty}$ converges to x .

Practical No. 2 –Continuous Functions on Metric Spaces

1. Which of the following subset of \mathbb{R}^2 are open? Justify.
 - a) $A = \{(x, y) \in \mathbb{R}^2 \mid x \text{ and } y \text{ are rationals}\}$
 - b) $B = \{(x, y) \in \mathbb{R}^2 \mid x \text{ and } y \text{ are both rationals and irrationals}\}$
2. If A and B are open subsets of \mathbb{R} then show that $A \times B$ is an open subset of \mathbb{R}^2 .
3. Let f and g be two real valued continuous functions on metric space M , let B be the set of all $X \in M$ such that $f(x) \geq g(x)$. prove B is closed.
4. Give an example of a sequence A_1, A_2, \dots Of non empty closed subsets of \mathbb{R} such that both of the following conditions hold :
 - a) $A_1 \supset A_2 \supset A_3 \dots$
 - b) $\bigcap_{n=1}^{\infty} A_n = \emptyset$.
5. Show that \mathbb{R} and R_d are not homeomorphic to each other.
6. Prove that $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ with absolute value metrics is homeomorphic to $[-1, 1]$.
7. Let R_d be a discrete metric space and $a \in R_d$. then find $S(a, 1)$ and hence using this show $\{a\}$ is both open and closed in R_d

Practical No. 3 – Connectedness and Completeness of Metric Spaces

1. If A is connected subset of metric space M and if $A \subset B \subset \bar{A}$ then prove that B is connected.
2. Show that $(0, 1)$ is not complete but connected subset of usual metric space \mathbb{R} .
3. Let A be a subset of \mathbb{I}^2 space consisting of the points $e_1 = (1, 0, 0, \dots)$, $e_2 = (0, 1, 0, \dots)$, $e_3 = (0, 0, 1, \dots)$ then show that A is bounded subset of \mathbb{I}^2 but it is not totally bounded.
4. If A is totally bounded subset of metric space $\langle M, \rho \rangle$ then prove that \bar{A} is totally bounded subset of M .
5. Let $\langle M, \rho \rangle$ be a metric space If $T: M \rightarrow M$ is a contraction on M then prove that T is continuous on M .
6. Prove that a connected subset of a discrete metric space R_d is complete.
7. Prove that R^2 is a complete metric space.

Practical No. 4 – Compact Metric Spaces

1. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \sin x \forall x \in \mathbb{R}$. Examine whether $f(x)$ is uniformly continuous or not.
2. Show that $f(x) = x^2 \forall x \in [0, 1]$ is uniformly continuous on $[0, 1]$ using definition of uniformly continuous function.
3. Show that every finite subset E of any metric space $\langle M, \rho \rangle$ is compact.
4. Give example of
 - a) Complete, compact and connected metric space.
 - b) Complete, compact but not connected metric space.
5. Let, f be a continuous function from the compact metric space M_1 into the metric space M_2 . Then prove that range $f(M_1)$ of f is bounded subset of M_2 .
6. Give an example of the function f from metric space M_1 to metric space M_2 such that f is continuous and one - one but f^{-1} is not necessarily continuous.

Practical No. 5 Riemann Integration

- Let $f(x) = x^3$ defined on $[0, k]$. Show that $f(x) \in R[0, k]$ and evaluate $\int_0^k x^3 dx$.
- Show that the function defined as $f(x) = \frac{1}{z^n}$ where $\frac{1}{z^{n+1}} < x \leq \frac{1}{z^n}$, $n = 0, 1, 2, \dots$. $f(0) = 0$ is integrable on $[0, 1]$ and evaluate $\int_0^1 f(x) dx$.
- The function $f(x)$ defined on $\left[0, \frac{\pi}{4}\right]$ as

$$f(x) = \cos x, \quad x \text{ is rational in } \left[0, \frac{\pi}{4}\right]$$

$$= \sin x, \quad x \text{ is irrational in } \left[0, \frac{\pi}{4}\right]$$
 Show that $f(x) \notin R\left[0, \frac{\pi}{4}\right]$.
- A function defined on $[0, 1]$ as

$$f(x) = \frac{1}{a^{r-1}} \text{ if } \frac{1}{a^r} < x \leq \frac{1}{a^{r-1}}, \text{ where } a \text{ is an integer greater than } z \text{ and } r = 1, 2, 3, \dots$$
 Show that
 - $\int_0^1 f(x) dx$ exist.
 - $\int_0^1 f(x) dx = \frac{a}{a+1}$
- Evaluate $\lim_{n \rightarrow \infty} \frac{1}{n} \left[\frac{1^{235} + 2^{235} + 3^{235} + \dots + n^{235}}{n^{235}} \right]$
- Evaluate $\lim_{n \rightarrow \infty} \left[\frac{n^n}{n!} \right]^{\frac{1}{n}}$
- Using Riemann integral as a limit of sum evaluate $\int_1^2 (5x + 7) dx$.

Practical No. 6 Mean Value Theorems

- Using Mean Value Theorem, prove that $\frac{\pi^3}{18} \leq \int_0^\pi \frac{x^2}{5 + \cos x} dx \leq \frac{\pi^3}{12}$
- Show that $\frac{1}{2} \leq \int_0^{1/2} \frac{dx}{\sqrt{1-x^2+x^3}} \leq \frac{\pi}{6}$
- Show that

$$\frac{1}{\sqrt{2}} < \int_0^{1/\sqrt{2}} \frac{dx}{\sqrt{1-x^4}} < \frac{\pi}{4} \text{ for all } x \in \left[0, \frac{1}{\sqrt{2}}\right].$$
- Verify Bonnet's Mean Value Theorem on $[-1, 1]$ for the function $f(x) = e^x$ and $g(x) = x$.
- Show that $\lim_{n \rightarrow \infty} \int_0^1 \frac{nf(x)}{1+n^2x^2} dx = \frac{\pi}{2} f(0)$.
- Verify Weierstrass second Mean Value Theorem for $f(x) = x$ and $g(x) = \sin x$ defined on $[\pi, 2\pi]$.
- If $0 < a < b$ then show that
 - $\left| \int_a^b \sin(x^2) dx \right| \leq \frac{1}{a}$
 - $\left| \int_a^b \frac{\sin x}{x} dx \right| \leq \frac{2}{a}$

Practical No. 7 Improper Integrals

1. Test the convergence

$$a) \int_0^{\pi/2} \frac{\sin x}{x^p} dx \quad b) \int_0^2 \frac{\log x}{\sqrt{2-x}} dx \quad c) \int_0^1 \frac{dx}{x^{1/2}(1-x)^{1/2}}$$

2. Examine the convergence

$$a) \int_1^{\infty} \frac{dx}{x^{1/3}(1+x)^{1/2}} \quad b) \int_1^{\infty} \frac{dx}{x\sqrt{1+x^2}} \quad c) \int_1^{\infty} \frac{\cos x}{\sqrt{x^2+x}} dx$$

3. Using Cauchy's test show that $\int_0^{\infty} \frac{\sin x}{x} dx$ is convergent.

4. Show that the integral $\int_0^{\pi/2} \log(\sin x) dx$ is convergent and hence evaluate it.

5. Discuss the convergence of $\int_0^1 \log \sqrt{x} dx$ and hence evaluate it.

Practical No. 8 Legendre Polynomials

1. Express $x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre's polynomial.

2. Express $2 - 3x + 4x^2$ in terms of Legendre's polynomial.

3. Prove that a) $P_n(1) = 1$, b) $P_{2n+1}(0) = 0$.

4. Prove that $\int_0^{\pi} P_n(\cos \theta) \cos n\theta d\theta = \beta \left(n + \frac{1}{2}, \frac{1}{2} \right)$.

5. Prove that $\frac{1+z}{z\sqrt{1-2xz+z^2}} - \frac{1}{z} = \sum_{n=0}^{\infty} (P_n + P_{n+1})z^n$

6. Prove that $P_n \left(-\frac{1}{z} \right) = P_0 \left(-\frac{1}{z} \right) P_{2n} \left(\frac{1}{z} \right) + P_1 \left(-\frac{1}{z} \right) P_{2n-1} \left(\frac{1}{z} \right) + \dots$
 $+ P_{2n} \left(-\frac{1}{z} \right) P_0 \left(\frac{1}{z} \right)$.

T.Y.B.Sc Mathematical Practical Course MTH-358

Based on MTH-353, MTH-354

Index

Practical No.	Title of Practical
1	Normal Subgroup and Isomorphism Theorems for groups
2	Permutation Groups
3	Quotient rings and Isomorphisms of rings
4	Polynomial rings
5	Posets
6	Lattices
7	Ideals and Homomorphisms
8	Modular and Distributive Lattices

MTH-353: Modern Algebra

Practical No.-1: Normal Subgroup and Isomorphism Theorems for groups

1. If H is subgroup of group G and $N(H) = \{g \in G : gHg^{-1} = H\}$, then prove that
 - a) $N(H)$ is normal subgroup of G .
 - b) H is normal subgroup of $N(H)$.
2. Give an example of subgroups H, K of G such that H is normal in K and K normal in G but H is not normal in G .
3. a) Let $G = \{A : A \text{ is non-singular } 2 \times 2 \text{ matrix over } \mathbb{R}\}$, a group under usual matrix multiplication and $H = \{A \in G : |A| = 1\}$ a subgroup of G . Show that H is normal in G .
 b) Show that $\langle G, + \rangle$ cannot be isomorphic to $\langle Q^*, \cdot \rangle$ where $Q^* = Q - \{0\}$ and Q = the set of rationals.
4. If $G = (\mathbb{Z}, +)$ and $N = (3\mathbb{Z}, +)$ find the quotient group G/H .
5. a) Let $Z =$ group of integer under addition then $f : Z \rightarrow Z$ such that $f(n) = -n$. Show that f is homomorphism and hence automorphism.
 b) Let $f : G \rightarrow G$ be a homomorphism, suppose f commutes with every inner automorphism of G , Show that $K = \{x \in G : f^2(x) = f(x)\}$ is a normal subgroup of G .

Practical No.-2: Permutation Groups

1. Prepare a multiplication table of the permutations on $S = \{1, 2, 3\}$ and show that S_3 is a group under the operation of permutation multiplication.
2. Express the following permutation ρ of degree 9 into the product of transpositions and find its order. Also find whether the permutation is odd or even, where $\rho =$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 4 & 6 & 5 & 3 & 2 & 9 & 7 & 8 \end{pmatrix}.$$
3. Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 4 & 1 & 8 & 2 & 5 & 7 \end{pmatrix}$ be a permutation in S_8 .
 - i) Find order of σ^{-1}
 - ii) Express σ^{-1} into the product of transpositions.
4. If $\sigma = (1\ 3\ 5\ 4)(2\ 6\ 8)(9\ 7)$ and $\delta = (8\ 9\ 7\ 6)(5\ 4\ 1)(2\ 3)$, then find i) σ^{-1} and δ^{-1} ii) Order of σ , δ and $\sigma\delta$.
5. Does there exists a permutation ρ in S_8 such that $\rho\sigma\rho^{-1} = \mu$ if
 - i) $\sigma = (2\ 3)$ and $\mu = (1\ 5\ 7)$
 - ii) $\sigma = (1\ 3\ 4)(5\ 6)(2\ 7\ 8)$ and $\mu =$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 5 & 8 & 6 & 2 & 7 & 4 & 1 \end{pmatrix}.$$

Practical No.-3: Quotient rings and Isomorphisms of rings

- Show that the intersection of two ideal is an ideal. But union may not be.
 - Let a, b are commutative element of a ring R of characteristic 2. Show that

$$(a + b)^2 = a^2 + b^2 = (a - b)^2.$$
- If D is an integral domain and $na = 0$ for some $0 \neq a \in D$ and some integer $n \neq 0$, then show that the characteristic of D is finite.
 - Let L be left ideal of ring R and let $\lambda(L) = \{x \in R : xa = 0, \text{ for all } x \in L\}$, then show that $\lambda(L)$ is an ideal of R .
- Find all prime ideals and maximal ideals of $(\mathbb{Z}_{12}, +_{12}, \times_{12})$.
 - If R is division ring, then show that the centre $Z(R)$ of R is a field.
- Let $Z[i] = \{a + ib : a, b \in \mathbb{Z}\}$.
 - Show that $Z[i]$ is an integral domain.
 - Find the field of quotients of $Z[i]$.
- Let R be a ring with identity 1 and $f : R \rightarrow R'$ be a ring homomorphism. Show that
 - if R is an integral domain and $\ker(f) \neq 0$ then $f(1)$ is an identity element of R' .
 - if f is onto then $f(1)$ is an identity element of R' .
- Let $R^c = \{f : [0, 1] \rightarrow \mathbb{R} : f \text{ is continuous}\}$ be a ring under the operations $(f + g)(x) = f(x) + g(x)$ and $(fg)(x) = f(x)g(x)$ and $(\mathbb{R}, +, \cdot)$ be a ring of real under usual addition and multiplication. Show that $\theta : R^c \rightarrow (\mathbb{R}, +, \cdot)$ defined by $\theta(f) = f\left(\frac{1}{2}\right)$, for all $f \in R^c$, is onto ring homomorphism. Hence prove that $\{f \in R^c : f\left(\frac{1}{2}\right) = 0\}$ is a maximal ideal of R^c .

Practical No.-4: Polynomial rings

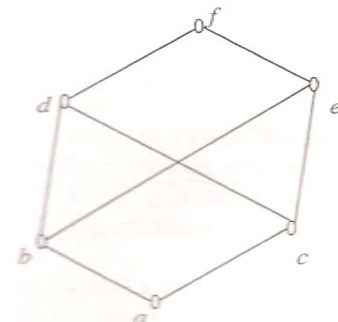
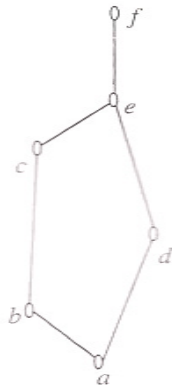
- Let $f(x) = 2x^3 + 4x^2 + 3x - 2$ and $g(x) = 3x^4 + 2x + 4$ be polynomials over a ring $(\mathbb{Z}_5, +_5, \times_5)$.
Find
 - $f(x) + g(x)$
 - $\deg(f(x)g(x))$
 - zeros of $f(x)$ in \mathbb{Z}_5 .
- Examine whether the polynomial $x^3 + 3x^2 + x - 4$ is irreducible over the field $(\mathbb{Z}_7, +_7, \times_7)$.
- Using Eisenstein's Criterion show that the following polynomials are irreducible over the field of rational numbers.
 - $x^4 - 4x + 2$
 - $x^3 - 9x + 15$
 - $7x^4 - 2x^3 + 6x^2 - 10x + 18$.
- Prove that the polynomial $1 + x + x^2 + x^3 + \dots + x^{p-1}$ is irreducible over the field of rational numbers, where p is a prime number.
- Show that $\frac{\mathbb{Z}_3[x]}{\langle x^2 + x + 1 \rangle}$ is not an integral domain.
- Show that $\langle x^2 + 1 \rangle$ is not a prime ideal of $\mathbb{Z}_2[x]$.

Practical No. 5 – Posets MTH-354 Lattice Theory

- 1) In any poset P , show that if $a, b, c \in P$, $0 < a$ for number a and $a < b$ and $b < c$ then $a < c$.
- 2) Prove that the two chains $S = \{0, \dots, \dots\}$ with respect to and $T = \{0, \dots, \dots\}$ with respect to are dually isomorphic.
- 3) Let A and B be two posets then show that $A \times B = \{(a, b) : a \in A, b \in B\}$ is poset under relation defined by $(a_1, b_1) \leq (a_2, b_2)$ iff $a_1 \leq a_2$ in A , $b_1 \leq b_2$ in B
- 4) Let S be set of even numbers up to 12. Define a relation on S as $a \leq b$ means a divides b . Prove that S is a poset under \leq . Draw poset diagram of S .
- 5) Let S be any set and L be a lattice. Let $T =$ set of functions from $S \rightarrow L$ Define relation ' \leq ' on T by $f \leq g$ iff $f(x) \leq g(x)$ for all $x \in S$. Then prove that $\langle T, \leq \rangle$ is a poset.

Practical No. 6 - Lattices

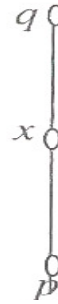
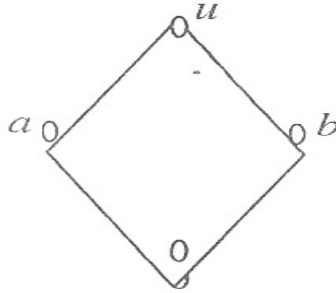
- 1) Show that a lattice L is a chain iff every non-empty subset of it is a sub lattice.
- 2) Let L be a lattice and $a, b \in L$ with $a \leq b$. Define $[a, b] = \{x \in L : a \leq x \leq b\}$. Show that $[a, b]$ is a sub lattice of L .
- 3) Draw diagram of the lattice L of all 16 factors of natural number 216 where $a \leq b$ means a divides b .
- 4) Determine which of the following are lattices



- 5) Give an example of smallest modular lattice which is not distributive.

Practical No. 7 – Ideals and Homomorphism

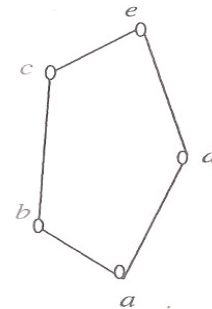
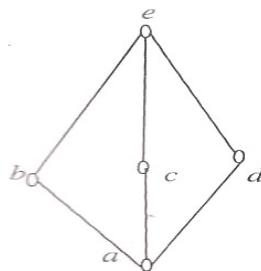
- 1) Let N be the Lattice of all natural numbers under divisibility. Show that $A = \{1, p, p^2, \dots\}$ where p is prime, forms an ideal of N .
- 2) Give an example to show that union of two ideals need not be an ideal.
- 3) Let I be a prime ideal of lattice L . show that $L - I$ is dual prime ideal.
- 4) Let I and M be lattices given as



- a) Define $f: I \rightarrow M$
Show that f is a homomorphism.
 - b) Define $g: M \rightarrow I$
such that g is neither a meet nor a join homomorphism
 - c) Define $h: I \rightarrow M$
show that h is a meet homomorphism but not a join homomorphism
- 5) In a finite lattice prove that every ideal is principal ideal.
 - 6) Show that in a complemented lattice the complement x' of an element is unique.

Practical No. 8 – Modular and Distributive Lattices

- 1) Determine which of the following lattices are distributive.



- 2) Prove that homomorphic image of distributive lattice is distributive.
- 3) Prove that homomorphic image of modular lattice is modular.
- 4) Show that lattice L is distributive if and only if $a, b, c \in L$ the inequality $a \wedge c \leq b \leq a \vee c$ then $(a \wedge b) \vee (b \wedge c) = (a \vee b) \wedge (b \vee c)$.
- 5) If a, b, c are elements of modular lattice L with greatest element u and if $a \vee b = (a \wedge b) \vee c = u$, then show that $a \vee (b \wedge c) = b \vee (c \wedge a) = c \vee (a \wedge b) = u$

T.Y.B.Sc Mathematical Practical Course MTH-359
Based on MTH-355(A) OR MTH-355(B) , MTH-356(A) OR
356(B)

Index

Practical No.	Title of Practical
1(A)	Basic concepts
2(A)	Expressions and conditional statements
3(A)	Decision making and Looping:
4(A)	Arrays and Functions
	OR
1(B)	Prime numbers and Diophantine Equations
2(B)	The theory of congruences
3(B)	Fermats Theorem and Perfect numbers
4(B)	Fibonacci numbers and Finite continued fractions
5(A)	Vector Functions
6(A)	Differential operator
7(A)	Vector Integration:
8(A)	Integral Theorem
	OR
5(B)	Fourier transforms
6(B)	Properties and Theorems of Fourier Transforms
7(B)	Transform :-
8(B)	Inverse Z- Transform

Practical No : 1(A) Basic concepts

1. Write a C program that will obtain the area and circumference of the circle given radius of the circle.
2. Write a C program to find the area of a triangle, given three sides.
3. Write a C program to find the simple interest, given principle, rate of interest and time.
4. Write a C program to read a five digit integer and print the reverse of its digits.
5. Write a C program to convert a given number of days into months and days.
6. A computer manufacturing company has the following monthly compensation policy to their sales persons :
Minimum base salary : 15000.00
Bonus for every computer sold : 2000.00
Commission on the total monthly sales : 3 per cent

Assume that the sales price of each computer is fixed at the beginning of every month.
Write a C program to compute a sales person's gross salary.

Practical No : 2(A) Expressions and conditional statements

1. Write a C program that determines whether a given integer is odd or even and displays the number and description on the same line.
2. Write a C program that determines whether a given integer is divisible by 3 or not and displays the number and description on the same line.
3. Write a C program that determines the roots of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$.
4. Write a C program to print the largest of the three numbers using nested if . . .else statement.
5. Write a program to check given alphabet is vowel or consonant.
6. Write a program to check whether given year is leap or not.
7. A company insures it's drivers in the following cases, if the driver is married, if the drivers is unmarried male & above 30 years of age, If the driver is unmarried female & above 25 years of age. Write a C-program.

Practical No : 3(A) Looping

1. Write a C program to find the sum of even natural numbers from 400 to 500.
2. Write a C program that determines whether a given integer is prime or not.
3. Write a C program to prepare multiplication table from 21 to 40.
4. Write a C program to generate and print first n Fibonacci numbers.
5. Write a C program to find the following sum

$$\text{Sum} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n}$$

6. Write a C program to get output by using for loop.

A

A B

A B C

A B C D

A B C D E

7. Write a program to find the LCM of two positive integers.

Practical No : 4(A) Arrays and Functions

1. Write a C program to sort N numbers in ascending order.
2. Write a C program to read two matrices and perform addition and subtraction of these matrices.
3. Write a C program to find the transpose of a given matrix.
4. Write a C program to read N integers (zero, positive, negative) into an array, and
 - i) find the sum of negative numbers.
 - ii) find the sum of positive numbers.
 - iii) find the average of all input numbers.

Output the various results computed with proper heading.

5. Given below is the list of marks obtained by a class of 50 students in an annual examination 43, 65, 51, 27, 79, 11, 56, 61, 82, 09, 25, 36, 07, 49, 55, 63, 74,

81, 49, 37, 40, 49, 16, 75, 87, 91, 33, 24, 58, 78, 65, 56, 76, 67,

45, 54, 36, 63, 12, 21, 73, 49, 51, 19, 39, 49, 68, 93, 85, 59.

Write a C program to count the number of students belonging to each of following groups of marks 0-9, 10-19, 20-29, . . . ,100.



Practical No.1(B) Prime numbers and Diophantine Equations

- 1 a) If $p \geq 5$ is a prime, then show that $p^2 + 2$ is composite.
 b) If $p \neq 5$ is an odd prime, prove that either $p^2 - 1$ or $p^2 + 1$ is divisible by 10.
- 2 Determine whether the integer 769 is prime by testing all primes $p \leq \sqrt{769}$ as possible divisors. Do the same for integer 1009.
- 3 a) Assuming that P_n is the n^{th} prime number, establish that the sum $\frac{1}{P_1} + \frac{1}{P_2} + \dots + \frac{1}{P_n}$ is never an integer.
 b) Show that the sum of twin primes P and $p + 2$ is divisible by 12, $P > 3$.
- 4 a) If $n > 1$, then show that $n!$ is never a perfect square.
 b) Find the values of $n \geq 1$, for which $n! + (n + 1)! + (n + 2)!$ is a perfect square.
- 5 a) Determine all solutions in the integers of the Diophantine equation $24x + 138y = 18$.
 b) Determine all solutions in the positive integers of the Diophantine equation $18x + 5y = 48$.

Practical No. 2(B) The theory of congruences

1. a) Assuming that 495 divides $273x49y5$ obtain the digits x and y .
 b) Show that the number 22125744515 is divisible by 37.
2. Solve the following linear congruences,
 i) $6x \equiv 15 \pmod{21}$
 ii) $34x \equiv 60 \pmod{98}$
3. Solve the following system of linear congruence by using CRT,
 $x \equiv 5 \pmod{6}, x \equiv 4 \pmod{11}, x \equiv 3 \pmod{17}$.
4. Solve the linear congruence $17x \equiv 3 \pmod{210}$ by solving the following system of linear congruence
 $17x \equiv 3 \pmod{2}, 17x \equiv 3 \pmod{3}, 17x \equiv 3 \pmod{5}, 17x \equiv 3 \pmod{7}$.
5. a) Use Fermat's method to factor 10541.
 b) Factor the number 14429 by searching for x , such that $x^2 - 14429$ is a perfect square

Practical No. 3(B) Fermats Theorem and Perfect numbers

- 1 a) Find the remainder when 11^{104} is divided by 17.
b) If $7 \nmid a$, prove that either $a^3 + 1$ or $a^3 - 1$ is divisible by 7.
- 2 a) Find the remainder when $4(29!) + 5!$ is divided by 31.
b) Show that $18! \equiv -1 \pmod{437}$
- 3 a) Show that the integer $n = 2^{10}(2^{11} - 1)$ is not perfect number.
b) If n is even perfect number then prove that $\sum_{d|n} \frac{1}{d} = 2$.
- 4 a) Prove that the Mersenne number M_{19} is prime. Hence show that $n = 2^{18}(2^{19} - 1)$ is perfect.
b) Find the Smallest Divisor of M_{11}
- 5 a) From the congruence $5 \cdot 2^7 \equiv -1 \pmod{641}$, deduce that $2^{32} + 1 \equiv 0 \pmod{641}$.
Hence $641 \mid F_5$.
b) For $n \geq 2$, show that the last digit of Fermat Number $F_n = 2^{2^n} + 1$ is 7.

Practical No. 4(B) Fibonacci numbers and Finite continued fractions

- 1 a) Evaluate $\gcd(u_9, u_{12}), \gcd(u_{15}, u_{20}), \gcd(u_{24}, u_{36})$.
b) Find the Fibonacci numbers that divides both u_{24} and u_{36} .
- 2 For primes $P = 7, 11, 13, 17$ verify that either u_{P-1} or u_{P+1} is divisible by P .
- 3 a) Express $\frac{187}{57}$ as finite simple continued fraction.
b) Express $\frac{-19}{51}$ as finite simple continued fraction.
- 4 a) Represent $[0:3, 1, 2, 3]$ as an odd number of partial denominators.
b) Represent $[-1:2, 1, 6, 1]$ as an odd number of partial denominators.
- 5 By using simple continued fraction, solve the Diophantine equation $19x + 51y = 1$.

Practical No .5A Vector Function

1. If $\vec{r} = \vec{a}e^{kt} + \vec{b}e^{-kt}$ where \vec{a} and \vec{b} are constant vectors and k is a constant scalar, show that $\ddot{\vec{r}} = k^2\vec{r}$.
2. Find the unit tangent vector and the curvature at a point $P(x, y, z)$ on the curve $x = a \cos \theta, y = a \sin \theta, z = a \theta \tan \alpha$, where a and α are constants.
3. A particle moves along the curve $\vec{r} = 2t^2\vec{i} + (t^2 - 4t)\vec{j} + (3t - 5)\vec{k}$. Obtain the components of its velocity and acceleration at $t = 1$ along the direction $\vec{i} - 3\vec{j} + 2\vec{k}$.
4. Find the acute angle between the tangents to the curve $\vec{r} = t^2\vec{i} - 2t\vec{j} + t^3\vec{k}$ at the points $t = 1$ and $t = 2$.
5. If $\vec{r} = \frac{a}{2}(x + y)\vec{i} + \frac{b}{2}(x - y)\vec{j} + xy\vec{k}$, find $\left[\frac{\partial \vec{r}}{\partial x}, \frac{\partial \vec{r}}{\partial y}, \frac{\partial^2 \vec{r}}{\partial x^2}\right]$ and $\left[\frac{\partial \vec{r}}{\partial x}, \frac{\partial \vec{r}}{\partial y}, \frac{\partial^2 \vec{r}}{\partial x \partial y}\right]$

Practical No .6A Differential operators

1. Find the scalar function ϕ , if $\nabla\phi = (y^2 - 2xyz^3)\vec{i} + (3 + 2xy - x^2z^3)\vec{j} + (6z^3 - 3x^2yz^2)\vec{k}$ & $\phi = (1, 1, 2) = 0$.
2. Find the equation of tangent plane & the normal line to the surface $xy + yz + zx = 7$ at $(1, 1, 3)$.
3. Find directional derivative of $\phi = xy + yz + zx$ at pt. the $(1, 2, 0)$ in the direction of the vector $\vec{i} + 2\vec{j} - 2\vec{k}$.
4. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then show that $\phi = \frac{1}{r}$ is harmonic function.
5. Find constants a, b, c so that $\vec{V} = (-4x - 3y + az)\vec{i} + (bx + 3y + 5z)\vec{j} + (4x + cy + 3z)\vec{k}$ is irrotational.

Practical No .7A Vecteor Integration

1. If $\vec{f} = (2y + 3)\vec{i} + xz\vec{j} + (yz - x)\vec{k}$, Evaluate, $\int_C \vec{f} \cdot d\vec{r}$ along the path C.

(1) $x = 2t^2, y = t, z = t^3$ from $t = 0$ to $t = 1$

(2) the $\int_C \vec{f} \cdot d\vec{r}$ straight line from $(0, 0, 0)$ to $(0, 0, 1)$ then to $(0, 1, 1)$ & then to $(2, 1, 1)$

(3) the straight line joining $(0, 0, 0)$ & $(2, 1, 1)$.

2. $\vec{f} = (y^2z^3\cos x - 4x^3)\vec{i} + (2z^3y\sin x)\vec{j} + (3y^2z^2\sin x - x)\vec{k}$ is a conservative force field.

find the scalar potential ϕ associated with \vec{f} .

3. If $\vec{f} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ evaluate $\iint_S \vec{f} \cdot \vec{n} \, ds$, where S is the surface of cube bounded by

$$x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$$

4. If $\vec{f} = (2x^2 - 3z)\vec{i} - 2xy\vec{j} - 4x\vec{k}$ evaluate $\iiint_V \nabla \cdot \vec{f} \, dv$,

where V is closed region bounded by the plane $x = 0, y = 0, z = 0$ & $2x + 2y + z = 4$

Practical No .8A Integral Theorem

1. Using Green theorem evaluate $\oint_C (2xy - x^2)dx + (x + y^2)dy$, where C is the closed curve of the

region bounded by $y = x^2, y^2 = x$.

2. Using Green's theorem, evaluate $\oint_C (e^{-x} \sin y \, dx + e^{-x} \cos y \, dy)$, where C is the rectangle formed

by $x = 0, x = \pi, y = 0, y = \frac{\pi}{2}$.

3. Verify the divergence theorem for the function $\vec{f} = 4xz\vec{i} - y^2\vec{j} + 4z\vec{k}$ over a unit cube bounded by

$$x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.$$

4. Verify Stoke's theorem for the vector field $\vec{f} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the area in the plane $z = 0$

bounded by $x = 0, y = 0$ and $x^2 + y^2 = 1$.

Practical No. 5B Fourier transforms

1. By considering Fourier sine and cosine transform for

$$f(x) = e^{-mx} \quad (m \geq 0), \quad \text{prove that}$$

$$\text{i) } \int_0^{\infty} \frac{s \sin sx}{s^2 + m^2} ds = \frac{\pi}{2} e^{-mx}, \quad m > 0, x > 0 \quad \text{and}$$

$$\text{ii) } \int_0^{\infty} \frac{\cos sx}{s^2 + m^2} ds = \frac{\pi}{2m} e^{-mx}, \quad m > 0, x > 0$$

$$2 \text{ Find Fourier transform of } f(x) = \begin{cases} 1-x^2 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$$

3. Find Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$ and use it to evaluate

$$\int_0^{\infty} \tan^{-1} \frac{x}{a} \sin x dx.$$

4. Using inverse sine transform, find $f(x)$ if $F_s(\lambda) = \frac{1}{\lambda} e^{-a\lambda}$

5. Solve the following integral equation

$$\int_0^{\infty} f(x) \sin \lambda x dx = \begin{cases} 1 - \lambda, & 0 \leq \lambda \leq 1 \\ 0, & \lambda \geq 1 \end{cases}$$

Practical No. 6B Properties and Theorems of Fourier Transforms

- 1 Find the finite Fourier sine and cosine transform of

$$f(x) = lx - x^2$$

2. Find $f(x)$, if $F_s[f(n)] = \frac{2l^3}{n^3\pi^3} (1 - \cos n\pi)$, where $0 \leq x \leq l$

3. Solve the equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$, $0 < x < \infty, t > 0$

Subject to the boundary condition $u(0, t) = f(t)$, $t > 0$

and $u(x, 0) = 0$, $x > 0$

4. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < \pi, t > 0$

Subject to the boundary condition

a) $u(x, 0) = 1$, $0 < x < \pi$,

b) $u(0, t) = u(\pi, t)$, $0 < x < \pi, t > 0$ using appropriate transform.

Practical No. 7B Z-Transform

1 Find

$$z\{f(k)\} \text{ if } f(k) = \begin{cases} 2^k & , k \geq 0 \\ \left(\frac{1}{3}\right)^k & , k < 0 \end{cases}$$

2. Find z-Transform of $f(k) = k^2 e^{-3k}$, $k \geq 0$
3. find z- Transform of $f(x) = 3^k [\cos (2k+3)]$
4. Verify convolution theorem for $f_1(k) = k$ and $f_2(k) = k$
5. Find z-transform. of $\left\{ \frac{\sin ak}{k} \right\}$ for $k \geq 0$

Practical No. 8B Inverse Z- Transform

1. Find the inverse Z-Transform of

$$F(Z) = \frac{1}{(Z-3)(Z-2)} \text{ if } |z| > 3$$

2. Find the inverse Z-Transform of $\left[\frac{z^2}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)} \right]$ if $\frac{1}{3} < |z| < \frac{1}{2}$ Find $z^{-1} \left(\frac{z^2}{z^2+1} \right)$
3. Find the sequence f if $F(e^{i\theta}) = \cos 3\theta$.
4. Find f(k) given

$$12f(k+2) - 7f(k+1) + f(k) = 0 \quad ; k \geq 0$$
 and $f(0) = 0; f(1) = 3$
5. Find $z^{-1} \left(\frac{z^2}{z^2+1} \right)$ by using inversion integral method

SEMESTER –II

MTH -361 : Measure and Integrations Theory

Unit 1: Measurable Sets

Periods -15, Marks -15

- 1.1 Length of open and closed sets.
- 1.2 Inner and outer measure of a sets.
- 1.3 Measurable sets
- 1.4 Properties of measurable sets.

Unit 2: Measurable functions

Periods -15, Marks -15

- 2.1 Real valued measurable functions
- 2.2 Sequence of measurable functions.
- 2.3 Measurable partition , Refinement , Lower and Upper Lebesgue sum , Lower and Upper Lebesgue integrals
- 2.4 Existence of Lebesgue integral for bounded function.

Unit 3: Lebesgue integral for bounded function and unbounded functi

Periobs -15, Marks -15

- 3.1 Properties of Lebesgue integral for bounded measurable functions.
- 3.2 Truncating function $^n f$
- 3.3 Positive and negative part of a function
- 3.4 Definition and properties of $\int_E f$ where f is non negative valued function in $L[a, b]$

Unit 4: Some fundamental theorems

Periods -15 , Marks 15

- 4.1 Lebesgue dominated convergence theorems
- 4.2 Fatou's Lemma
- 4.3 The metric space $L^2[a, b]$, square integrable function , Schwartz inequality , Minkowski inequality .

Recommended Book : [1] R.R. Goldberg, Methods of Real Analysis, John Weley and Sons, Inc. New York, 2nd Edition, 1976

Chapter 11 : 11.1,11.2,11.3, 11.4,11.5,11.6,11.7, 11.8, 11.9

Reference Books :

- 1) Measure and integration by G.D.Barra
- 2) Measure Theoty by K.D.Gupta.
- 3) Lebesgue Measure and integration by P.K.Jain >
- 4) Measure Theory by P.R.Halmos .

MTH-362: Methods of Real Analysis

Unit 1: Sequence of real numbers and Sequence of functions (Periods-15, Marks-15)

- 1.1. Definition of sequence and subsequence of real numbers. Convergence and divergence of sequence of real numbers
- 1.2. Monotone sequence of real numbers
- 1.3. Point wise convergence of sequence of functions
- 1.4. Uniform convergence of sequence of functions
- 1.5. Cauchy's criteria for uniform convergence of sequence of functions
- 1.6. Consequences of uniform convergence

Unit 2: Series of real numbers (Periods-15, Marks-15)

- 2.1. Convergence and divergence
- 2.2. Series with non-negative terms
- 2.3. Alternating Series
- 2.4. Conditional convergence and absolute convergence
- 2.5. Rearrangement of series
- 2.6. Test for absolute convergence
- 2.7. Series whose terms form non-increasing sequence

Unit 3: Series of functions (Periods-15, Marks-15)

- 3.1. Pointwise convergence of series of functions
- 3.2. Uniform convergence of series of functions
- 3.3. Integration and differentiation of series of functions
- 3.4. Abel's Summability

Unit 4: Fourier series in the range $[-\pi, \pi]$ (Periods-15, Marks-15)

- 4.1. Fourier series and Fourier coefficients
- 4.2. Dirichlet's condition of convergence (statement only)
- 4.3. Fourier series for even and odd functions
- 4.4. Sine and Cosine Series in half range

Recommended Books:

- 1) [1] R.R. Goldberg, Methods of Real Analysis, John Weley and Sons, Inc. New York, 2nd Edition, 1976.
 Unit 1:- 2.1, 2.3, 2.4, 2.6, 9.1, 9.2
 Unit 2:- 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7
 Unit 3:- 9.4, 9.5, 9.6
- 2) Laplace Transform and Fourier series, Schaum series (Unit – 4).

Reference Books:

- 1) S.C.Malik & Savita Arora, Mathematical Analysis (2010).
 - 2) Mathematical Analysis by S.K. Chatterjee.
-

MTH – 363 Linear Algebra**Unit 1: Vector Spaces****Periods – 15, Marks – 15**

- 1.1. Vector spaces, Subspaces, Examples.
- 1.2. Necessary and sufficient conditions for a subspace.
- 1.3. Addition, Intersection and union of subspaces.
- 1.4. Quotient space.
- 1.5. Linear span and properties.
- 1.6. Linear dependence and independence.

Unit 2: Basis and Dimensions**Periods – 15, Marks – 15**

- 2.1 Basis and dimension of finite dimensional vector spaces.
- 2.2 Co-ordinates of a vector.
- 2.3 Existence theorem, Invariant of elements in a basis.
- 2.4 Extension theorem.
- 2.5 Theorems on basis and dimensions.

Unit 3: Linear Transformations**Periods – 15, Marks – 15**

- 3.1 Range space and null space of linear transformations.
- 3.2 Rank and nullity theorem.
- 3.3 Vector space $L(V, W)$.
- 3.4 Algebra of linear transformations and theorems on isomorphism.
- 3.5 Invertible linear transformations.
- 3.6 Singular and non-singular linear transformations.
- 3.7 Representation of linear transformation by matrix.

Unit 4: Eigen values and Eigen vectors**Periods – 15, Marks – 15**

- 4.1 Matrix polynomial.
- 4.2 Characteristics polynomial and minimum polynomial.
- 4.3 Eigen values and Eigen vectors of linear transformation.
- 4.4 Similarity.
- 4.5 Diagonalisation of matrix.
- 4.6 Cayley Hamilton Theorem

Recommended Book:

- 1) V. K. Khanna and S. K. Bhambri, Course in Abstract Algebra (2013), Vikas Publishing House Pvt. Ltd. New Delhi.
- 2) K. P. Gupta J. K. Goyal, Advanced Course in Modern Algebra
- 3) A. R. Vasishtha and J.N. Sharma, Linear Algebra (2014), Krishna Publication, Meerut

Reference Books :

- 1) Theory and Problems of Linear Algebra, by S. Lipschutz, Schaum's outline series, SI(Metric) edition, (1987), McGraw Hill Book Company.
- 2) N. S. Gopalkrishnan, University Algebra(2015), New Age Int.Pvt.Ltd.

MTH-364 : Ordinary and Partial differential Equations

Unit 1: Exact Differential Equations

Period -15 Marks -15

- 1.1 Definition, Condition of exactness of a linear differential equation of order n.
- 1.2 Working rule for solving exact differential equations, examples of type –I.
- 1.3 Integrating factor, examples of type –II.
- 1.4 Exactness of non-linear equations, Solution by trial, examples of type –III.
- 1.5 Equation of the form $\frac{d^n y}{dx^n} = f(x)$, example on type – IV.
- 1.6 Equation of the form $\frac{d^2 y}{dx^2} = f(y)$, example on type – V.

Unit 2: Linear Differential Equations of second order

Period -15 Marks -15

- 2.1 The standard form of linear differential equations of second order.
- 2.2 Complete solution in term of one known integral belonging to C.F.
- 2.3 Rule for gating an integral belonging to C.F., Working rule for finding complete solution when n integral of C.F. is known.
- 2.4 Removal of first derivate , reduction to normal form, working rule for solving problems by using normal form.
- 2.5 Transformation of the equation by changing the independent variable, working rule.

Unit 3: Power series method

Period -15 Marks -15

- 3.1 Introduction
- 3.2 Some basic definition, ordinary and singular points.
- 3.3 Power series solution, Series solution about regular singular point $x=0$, Frobenius Method.
- 3.4 Frobenius method type –I
- 3.5 Frobenius method type –II
- 3.6 Frobenius method type –III based on.

Unit 4: Linear partial differential equations of the first order

Period -15 Marks -15

- 4.1 Introduction
- 4.2 Partial differential equation, definition, order and degree of partial differential equations,
- 4.3 Origins of first order partial differential equations, derivation of a partial differential equation by elimination of arbitrary constants and arbitrary functions.
- 4.4 Linear differential equation of the first order, complete integral, general integral, Lagrange's equations and Lagrange's method of solving $Pp+Qq=R$.
- 4.5 Integral surfaces passing through a given curve.
- 4.6 Surfaces orthogonal to a given system of surfaces.

Recommended book: M.D.Raisinghania, Ordinary and Partial differential Equations, S Chand Publication.

MTH-365(A): Optimization Techniques

Unit 1 : Linear programming problem (LPP) Periods – 15, Marks – 15

- 1.1 Formation of LPP
- 1.2 Solution of LPP by graphical method
- 1.3 Solution of LPP by simplex method
- 1.4 Artificial variable technique (Big M method)
- 1.5 Special cases in LPP : a) Unbounded solution b) Alternate solution c) No solution by graphical as well as simplex method
- 1.6 Degeneracy in simplex method and its resolution.

Unit 2 : Transportation problem (TP) Periods – 15, Marks – 15

- 2.1 Formation of TP. TP as LPP
- 2.2 Methods for finding IBFS : a) North –West corner rule. b) Matrix minima method (Least cost method) c) Vogel’s approximation method (VAM)
- 2.3 Optimality test and optimization of solution to TP by U-V method (MODI).
- 2.4 Special cases in TP : a) Alternate solution b) Maximization of TP c) Degeneracy in solving TP. d) Restricted transportation problems.

Unit 3 : Assignment problem (AP) Periods – 15, Marks – 15

- 3.1 Formation of Assignment problem AP as TP
- 3.2 Hungarian method for solving AP
- 3.3 Special cases in AP: a) Alternate solution b) Maximization of AP c) Restricted AP.

Unit 4: Game Theory Periods – 15, Marks – 15

- 4.1 Introduction
- 4.2 Competition situation and definition of game, Two person-zero sum game
- 4.3 Formation of Game
- 4.4 Pure and mixed strategies, value of a game
- 4.5 Maxmin and Minmax principles and saddle point 3+
- 4.6 Analytic solution of 2x2 game
- 4.7 Principle of dominance
- 4.8 Graphical solution of mx2 and 2xn games.

Reference Books:

1. Prem Kumar Gupta, Operation Research (2014, 7th Edition), S. Chand and Company pvt Ltd. New Delhi.
2. S. D. Sharma and Kedarnath Ramnath, Operation Research (2012), Meerut Publication.

MTH-365(B) Dynamics**Unit1: Kinematics****Period -15, Marks -15**

- 1.1 Displacement.
- 1.2 Motion in a straight line velocity and acceleration.
- 1.3 Motion in a plane velocity and acceleration.
- 1.4 Radial and transverse velocity and acceleration.
- 1.5 Angular velocity and acceleration.
- 1.6 Tangential and normal component of velocity and acceleration.

Unit 2: Rectilinear motion**Period -15, Marks -15**

- 2.1 Motion in a straight line with constant acceleration.
- 2.2 Motion in a train between two stations.
- 2.3 Simple harmonic motion.
- 2.4 Hook's law (a) Horizontal elastic strings (b) vertical elastic strings

Unit 3: Uniplaner motion**Period -15, Marks -15**

- 3.1 Projectile – Introduction.
- 3.2 Projectile – Equation of trajectory.
- 3.3 Projection to pass through a given point.
- 3.4 Envelope of the paths.

Unit 4: Central forces**Period -15, Marks -15**

- 4.1 Motion of a particle under central force.
- 4.2 Use of pedal co-ordinates and equation.
- 4.3 Apses.

Recommended Book:

- 1) M. Ray, A text book on Dynamics (2014), S.Chand and company, New Delhi

Reference Books:

- 1) M.D.Raisinghania, Dynamics (2010), S.Chand and company, New Delhi.
- 2) P.N. Chatterjee, Mechanics (2015), Rajhans publication.

MTH-366(A): Applied Numerical Methods

Unit 1: Simultaneous Linear Algebraic Equations. Period 15 Marks 15

- 1.1 Method of Factorization or triangularisation.
- 1.2 Crout's method.
- 1.3 Inverse of a matrix by Crout's method.
- 1.4 Gauss Seidel iteration method.
- 1.5 Relaxation method

Unit 2: Numerical Differentiation & Integration. Period 15 Marks 15

- 2.1 Numerical Differentiation.
- 2.2 Derivatives using Newton's Forward difference Formula.
- 2.3 Derivatives using Newton's Backward difference Formula.
- 2.4 Evaluation of double integrals using Trapezium rule.
- 2.5 Evaluation of double integrals using Simpson's rule.

Unit 3: Interpolation Period 15 Marks 15

- 3.1 Newton's divided difference formula.
- 3.2 Iterative method
- 3.3 Spine Interpolation & cubic spines.

Unit 4: Boundary value problems in ordinary and partial differential equations

Period 15 Marks 15

- 4.1 Introduction
- 4.2 Boundary value problems governed by second order ordinary differential equations.
- 4.3 Classification of Linear second order partial differential equations.
- 4.4 Finite difference method for Laplace & Poisson equations

Recommended Books

- 1) Dr. V N Vedamurthy & Dr. N Ch S N Iyengar, Numerical Methods (1998, 1st Edition), Vikas Publishing House Pvt Ltd
For Unit-1, all topics. For Unit -2, 2.1, 2.2, 2.3. For Unit-3, 3.1, 3.2, 3.3
- 2) S.R K. Iyengar & R, K Jain, Numerical Methods (2007, 6th Edition), New Edge International Pvt Ltd
For Unit -2, 2.4 For Unit-3, 3.4, 3.5 For Unit-4, all topics.

Reference Book. 1) P. Kandasamy, K, Thilagavathy & K. Gunavathy, Numerical Methods(2006), S. Chand & Co. Ltd.

MTH -366(B) : Differential Geometry**Unit 1: Curves in Spaces.****Periods -15 Marks -15**

- 1.1 Spaces curves , Parametric and vector representation of curves
- 1.2 Tangent line
- 1.3 Osculating plane and Normal Plane
- 1.4 Principal normal and binormal
- 1.5 Unit vectors ,t,n,b
- 1.6 Rectifying Plane
- 1.7 Curvature and Torsion
- 1.8 Radius of curvature and radius of Torsion
- 1.9 Screw Curvature ,Serret- Frenet formulae
- 1.10 Cutvature and torsion of curves , Helices

Unit 2:Cutvatures**Periods -15 Marks -15**

- 2.1 The circle of curvature.
- 2.2Osculating Sphere
- 2.3Involute and Evolute of a space Curves
- 2.4The spherical Indicatrices (images)
- 2.5 Bertrand Curves

Unit 3:Envelopes of Surfaces**Periods -15 Marks -15**

- 3.1 Parametric representation of surfaces
- 3.2 Tangent plane and Normal line at a point on the surface.
- 3.3 First fundamental form
- 3.4 Second fundamental form
- 3.5Envelpoes and Characteristics
- 3.6Edge of regression of relating to one and two parameter family of surfaces.

Unit 4: Developable Surfaces.**Periods -15 Marks -15**

- 4.1 Ruled surface (Developable and skew)
- 4.2 Devalopable surfaces
- 4.3Developable associated with space curves
- 4.4 Characterization of a developable Surface.

Recommended Book: 1) Dr.S.C.Mittal and D.C.Agrawal, Differential Geometry (2014, 38th Edition, Krishna Prakashan Mandir

Chapter 1 : 2,5,7 to 13 ,15,16,18,19,20.

Chapter 2: 1,3,4,5,6,7,10

Chapter 3: 9 to 14

References Books 1. M.L.Khanna Jaiprakash Nath and Company, Differential Geometry

2. P.P.Gupta , G.S.Malik. and S.K.Pundir, Differential Geometry, Pragati Prakashan

3. Bansilal, Three dimensional Cifferential Geometry (1976), Atmaram and Sons

SEMESTER -II**T.Y.B.Sc Mathematical Practical Course MTH-367****Based on MTH-361, MTH-362**

Index

Practical No.	Title of Practical
1	Measurable Sets
2	Measurable functions
3	Lebesgue integral for bounded and unbounded functions
4	Some fundamental Theorems
5	Sequence of real numbers and sequences of functions
6	Series of real numbers
7	Pointwise convergence and uniformly convergence of series of functions
8	Legendre Polynomials

MTH -361 – Lebesgue Integration Practcal

Practcal No.1 Measurable Sets

- 1) Show that Cantors set is measurable and has measure zero.
- 2) I) Show that $S = \bigcup_{n=1}^{\infty} \left[n - 3\frac{1}{n}, n \right]$ is measurable and find its measure.
 II) Find the length of $S = \bigcup_{n=1}^{\infty} \left\{ x : \frac{1}{3^k} \leq x \leq \frac{1}{3^{k-1}} \right\}$
- 3) I) Prove that a set consisting of one point is measurable and its measure is zero.
 II) If $E \subset [a, b]$ and $\bar{m}E = 0$ then prove that E is measurable and $mE = 0$
- 4) If E_1 and E_2 are measurable subset of $[a, b]$ then prove that the symmetric difference of E_1 and E_2 is also measurable.
- 5) If E is measurable subset of $[a, b]$ then prove that $\bar{m}A = \bar{m}(A \cap E) + \bar{m}(A \cap E^c)$

Practical No.2 Measurable functions

- 1) Show that a subset E of $[a, b]$ is measurable if and only if its characteristics function χ_E is measurable .
- 2) If $f(x) = \frac{1}{x}, 0 < x < 1$
 $= 5, \text{if } x = 0$
 $= 7, \text{if } x = 1$ then prove that f is measurable .
- 3) If f is measurable function on $[a, b]$ and if $c \in \mathbb{R}$ then prove that the function $c + f$ and cf is measurable.
- 4) Let $f(x) = 2$ if $0 \leq x < 1$
 $= 4$ if $1 \leq x < 2$
 $= 3$ if $2 \leq x < 3$
 $= 2$ if $3 \leq x \leq 4$
 I) For $k = 2, 3, 4$,Let E_k be the inverse image under f of $[k, k+1]$, show that $P = \{E_2, E_3, E_4\}$ is measurable partition of $[0, 4]$
 II) Calculate $U[f, P]$ and $L[f, P]$
- 5) Let $E_1, E_2, E_3, \dots, E_n$ be measurable subset of $[0, 1]$, if each point of $[0, 1]$ belongs to at least three of the sets , show that at least one of the set has measure $\geq \frac{3}{n}$

Practical No.3 Lebesgue integral for bounded and unbounded functions

1) a) If $f(x) = \log \frac{1}{x}$ for $0 < x \leq 1$, then find $\int f^2$

b) If $f(x) = \frac{1}{2} + \sin x$ for $0 \leq x \leq 2\pi$ then find f^+ and f^-

2) If $f(x) = \frac{1}{\sqrt[3]{x}}$ if $0 < x \leq 1$

$= 0$, if $x = 0$ then find the function f^+ and prove that f is Lebesgue integrable on $[0,1]$

3) Show that if $f(x) = \frac{1}{x}$ if $0 < x \leq 1$

$= 19$ if $x = 0$ then prove that f is not Lebesgue integrable on $[0,1]$

4. If $f(x) = \frac{1}{x^p}$ for $0 < x \leq 1$ prove that $f \in L[0,1]$ if $p < 1$ and $L \int_0^1 f = \frac{1}{1-p}$

5. If $f(x) = x^2 - 1$ for $-2 \leq x \leq 2$, find functions f^+ and f^-

Practical No.4 Some fundamental Theorems

1) Using Lebesgue dominated convergence theorem evaluate $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx$,

where

$$f_n(x) = \frac{n^{\frac{3}{2}} - x^{\frac{3}{2}}}{1 + n^2 + x^2}, \quad 0 \leq x \leq 1, \quad n = 1, 2, 3, \dots$$

2) If $f \in L[a,b]$ and $f(x) = \int_a^x f(t) dt$ for $a \leq x \leq b$. Prove that f is continuous on $[a,b]$

3) Show that $C[a,b] \subset L^2[a,b]$

4) If f is square integrable on $[a,b]$, then prove that $f \in L^1[a,b]$.

Practical No.5 Sequence of real numbers and sequences of functions

- If $\{x_n\}_{n=1}^{\infty}$ is sequence of real numbers which converges to L then show that $\{x_n^2\}_{n=1}^{\infty}$ Converges to L^2
 - $\{a_n\}_{n=1}^{\infty}$ is a Cauchy sequence of real numbers then show that $\{a_n\}$ is also Cauchy.
- Discuss the convergence of sequence $\left\{\left(1 + \frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$
- Let $f_n(x) = \frac{x^n}{1+x^n}$, $0 \leq x \leq 1$. Show that $\{f_n\}_{n=1}^{\infty}$ converges point wise on $[0, 1]$, if $\lim f_n(x) = f(x)$. Does there $N \in \mathbb{N}$ Such that $|f_n(x) - f(x)| < \frac{1}{4}$, for all $n \in \mathbb{N}$ for all $x \in [0, 1]$.
 - $\{f_n\}_{n=1}^{\infty}$ is uniformly convergent in any finite interval $f_n(x) = \frac{n}{n+x}$, $n \geq 0$
- Let $f_n(x) = \frac{\sin nx}{n}$, $0 \leq x \leq 1$. Show that $\{f_n\}_{n=1}^{\infty}$ converges uniformly to 0 but that $\{f_n\}_{n=1}^{\infty}$ does not converges even point wise to 0 on $[0, 1]$.
- Let $f_n(x) = \frac{nx}{1+n^2x^2}$, $x \in \mathbb{R}$. Show that $\{f_n\}_{n=1}^{\infty}$ is not uniformly converges in $[0, 1]$ although it converges point wise to 0.

Practical No.6 Series of real numbers

- Discuss the convergence of the series
 - $1 + x + x^2 + x^3 + x^4 + \dots$
 - $2^{\frac{1}{1}} - 2^{\frac{1}{2}} + 2^{\frac{1}{3}} - 2^{\frac{1}{4}} + 2^{\frac{1}{5}} + \dots$
 - $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{n}{2n-1}\right)$
- Examine the convergence of the series
 - $\sum_{n=1}^{\infty} \frac{(n+1)}{n^{10}(n+2)}$
 - $\sum (-1)^{n+1} \left(\frac{1}{n^p}\right)$, if $p > 0$
 - $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{x+n}$, $x \notin \mathbb{N}$
- Examine the convergence of the series
 - $\sum_{n=1}^{\infty} \frac{3}{4+2^n}$
 - $\sum_{n=1}^{\infty} \frac{1}{(\log n)^n}$
- Discuss the convergence of the series
 - $\sum 2^n \frac{n!}{n^n}$
 - $\sum x^n \frac{n!}{n^n}$
- Examine the convergence of
 - $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{-n^2}$

b. $\sum_{n=1}^{\infty} \frac{\left(1+\frac{1}{n}\right)^{2n}}{e^n}$

Practical No. 7 Pointwise convergence and uniformly convergence of series of functions.

1. Show that the following series are uniformly convergent for all values of x

a) $\sum_{n=1}^{\infty} \frac{\sin(x^2+n^2x)}{n(n+2)}$

b) $\sum_{n=1}^{\infty} \frac{\cos(x^2+n^2x)}{n(n^2+2)}$

2. Test the uniform convergence of the series $\sum_{n=1}^{\infty} xe^{-nx}$ on $[0, 1]$

3. Examine by term by term integration $s_n(x) = nxe^{-nx^2}$ in $[0, 1]$.

4. a) Without finding the sum $f(x)$ of the series

$$1 + \frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!} + \dots + \frac{x^{2n}}{n!} + \dots \quad (-\infty < x < \infty)$$

Show that $f'(x) = 2xf(x)$, $(-\infty < x < \infty)$

b) Show that $\frac{d}{dx} \left\{ \sum_{n=1}^{\infty} \frac{\sin nx}{n^3} \right\} = \sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$

5. Show that $\sum_{n=1}^{\infty} \frac{x}{n^p+n^q x^2}$ is uniformly convergent for all x and $p+q > 2$.

Practical No. 8 Fourier series

1. If f is bounded and integrable on $[-\pi, \pi]$ and if a_n, b_n are its Fourier coefficient then

$$\sum_{n=1}^{\infty} (a_n^2 + b_n^2) \text{ converges.}$$

2. Let $f(x) = -\cos x$ for $-\pi < x < 0$

$$= \cos x \text{ for } 0 < x < \pi$$

Show that the Fourier series which converges to $f(x)$ is

$$\frac{\pi}{4} \left(\frac{2}{1.3} \sin 2x + \frac{4}{3.2} \sin 4x + \frac{6}{5.7} \sin 6x + \dots \right)$$

3. Obtain the Fourier series of the function $f(x) = |x|$ in $[-\pi, \pi]$.

4. Find half range sine and cosine series for $f(x) = x$ in $[0, \pi]$

5. Find Fourier series for $f(x) = x + x^2$ in $[-\pi, \pi]$ and $f(x) = \pi^2$ when $x = \pm\pi$

$$\text{Deduce that } \frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$$

T.Y.B.Sc Mathematical Practical Course MTH-368

Based on MTH-363, MTH-364

Index

Practical No.	Title of Practical
1	Vector space, Subspace, Linearly Dependence and Independence
2	Basis and Dimensions
3	Linear Transformations
4	Eigen values and eigen vectors
5	Exact Differential Equations
6	Linear Differential Equations of second order
7	Power series method
8	Linear partial differential equations of the first order

MTH 363 – Linear Algebra**Practical No 1 Vector space, Subspace, Linearly Dependence and Independence**

- Let V be the set of all ordered pairs (p, q) of real numbers. Examine each of the following V is a vector space over R or not :
 - $(p, q) + (p', q') = (0, q + q'), \alpha(p, q) = (\alpha p, \alpha q)$.
 - $(p, q) + (p', q') = (p + p', q + q'), \alpha(p, q) = (0, \alpha q)$.
 - $(p, q) + (p', q') = (p + p', q + q'), \alpha(p, q) = (\alpha^2 p, \alpha^2 q)$.
- If $V_3(R)$ be the vector space of all ordered triads (x, y, z) . Determine which of the following subsets of $V_3(R)$ are subspaces :
 - $W = \{(x, y, z) / x, y, z \in R \text{ and } x - 3y + 4z = 0\}$.
 - $W = \{(x, y, z) / x, y, z \in R \text{ and } x, y, z \text{ are rational}\}$.
- Let V be the vector space of all functions from R to R .
Let $V_1 = \{f \in V / f(-x) = f(x), \forall x \in R\}$ and
 $V_2 = \{f \in V / f(-x) = -f(x), \forall x \in R\}$. Prove that V_1, V_2 are subspaces of V .
- Examine the set of vectors $\{(1, 2, -3), (1, -3, 2), (2, -1, 5)\}$ for linearly dependent or linearly independent.
- If u, v, w are linearly independent vectors in $V(f)$, show that the vectors
 - $u + v, v + w, w + u$ are linearly independent
 - $u + 3v - 2w, 2u + v - w, 3u + v + w$ are linearly independent
- Which of the following set of polynomials in $P_2(x)$ are linearly independent:
 - $1 + x + 2x^2, 2 - x + x^2, 4 + 5x + x^2$.
 - $2 - x + 4x^2, 3 + 6x + 2x^2, 2 + 10x - 4x^2$.
- Prove that the four vectors $(1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 1)$ in R^3 form a linearly dependent set but any three of these are linearly independent.

Practical No 2 Basis and Dimensions

- Find the coordinate vector of $v = (3, 5, -2)$ relative to the basis of $e_1 = (1, 1, 1), e_2 = (0, 2, 3), e_3 = (0, 2, -1)$.
- If W is the subspace of $V_4(R)$ generated by the vectors $(3, 8, -3, -5), (1, -2, 5, -3), (2, 3, 1, -4)$ then
 - find the basis and dimension of W .
 - Extend the basis of W to a basis of $V_4(R)$.
- Find the basis and dimension of the solution space W of the following system of equations :

$$\begin{aligned}x + 2y - 4z + 3s - t &= 0 \\x + 2y - 2z + 2s + t &= 0 \\2x + 4y - 2z + 3s + 4t &= 0\end{aligned}$$

- If V_1 and V_2 are the subspaces of a vector space R^3 generated by the sets $S_1 = \{(1, 2, 3), (0, 1, 2), (3, 2, 1)\}$ and $S_2 = \{(1, -2, 3), (-1, 1, -2), (1, -3, 4)\}$ respectively. Determine
 - $\dim(V_1 + V_2)$
 - $\dim(V_1 \cap V_2)$.
- Reduce the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 2 & 3 & 5 & 8 \\ 3 & 4 & 5 & 8 \\ 1 & 1 & 2 & 3 \end{bmatrix}$$

to row echelon form and hence find the dimension of the row space of the matrix A .

- Let W_1 and W_2 be two subspaces of R^4 given by $W_1 = \{(a, b, c, d) : b + d = 2c\}$ and $W_2 = \{(a, b, c, d) : a = b, b = 2c\}$. Find the basis and dimension of
 - W_1
 - W_2
 - $W_1 + W_2$
 - $W_1 \cap W_2$
- Show that the set of vectors $(0, 1, -1), (1, 1, 0)$ and $(1, 0, 2)$ is a basis of $R^3(R)$
- Given two linearly independent vectors $(1, 1, 1, 1)$ and $(1, 2, 1, 2)$. Find the basis of $V_4(R)$ that includes these two vectors.

Practical No 3 Linear Transformations

- Let $f: R^4 \rightarrow R^3$ be the linear map defined by $f(x, y, z, t) = (x - y + z + t, x + 2z - t, x + y + 2z - 3t)$. Find a basis and dimension of the image of f .
- Show that the map $T: V_2(R) \rightarrow V_1(R)$ defined as $T(a, b) = (a + b, a - b, b)$ is a linear transformation from $V_2(R)$ to $V_1(R)$. Find the range, rank, null space and nullity of T . Verify that $\text{rank}(T) + \text{nullity}(T) = \dim V_2(R)$.
- Let T be a linear operator on R^3 defined by $T(x, y, z) = (2y + z, x - 4y, 3x)$. Find the matrix of T in the basis $B = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$.
- Let T be a linear operator on R^3 defined by $T(x, y, z) = (3x + z, -2x + y, -x + 2y + 4z)$. Prove that T is invertible and find the formula for T^{-1} .
- Show that the linear mapping $T: V_3 \rightarrow V_3$ defined by $T(a, b, c) = (a + b + c, b + c, c)$ is non-singular and find its inverse.

6. Find a linear transformation $T : R^3 \rightarrow R^3$ such that $T(1, 1, 1) = (1, 1, 1), T(1, 2, 3) = (-1, -2, -3), T(1, 1, 2) = (2, 2, 4)$.
7. Let T be a linear operator on R^2 defined by $T(x, y) = (4x - 2y, 2x + y)$.
Compute the matrix of T relative to the basis $\{(1, 1), (-1, 0)\}$.
8. Let $T : R^3 \rightarrow R^2$ be a linear transformation defined by
 $T(x, y, z) = (2x + y - z, 3x - 2y + 4z)$. Obtain the matrix of T in the following bases of R^3 and R^2 .
 $B_1 = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}, B_2 = \{(1, 3), (1, 4)\}$.

Practical No 4 Eigen values and eigen vectors

1. Find the Eigen values and associated eigen vectors of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.
2. Find the characteristics roots, their corresponding vectors and the basis for the vector space of a matrix

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

3. Verify Cayley-Hamilton Theorem for the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \text{ and hence obtain } A^{-1}.$$

4. Find the eigen equation of the equation of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \text{ and verify it is satisfied by } A. \text{ Hence obtain } A^{-1}.$$

5. Show that the matrix

$$A = \begin{bmatrix} 1 & -1 & 4 \\ -3 & 2 & 1 \\ 2 & 1 & -1 \end{bmatrix} \text{ is diagonalizable.}$$

6. Find the matrix P , if it exists, which diagonalizes A , where $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 1 & 3 \end{bmatrix}$

7. Find the minimum polynomial of the matrix

$$A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}.$$

8. Find the characteristics polynomial and the minimum polynomial of

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

MTH-364 : Ordinary and Partial differential Equations

Practical No.5: Exact Differential Equations

- 1) Solve $(1 + x + x^2) \frac{d^3y}{dx^3} + (3 + 6x) \frac{d^2y}{dx^2} + 6 \frac{dy}{dx} = 0$
- 2) Find m if x^m is an integrating factor of $x^5 \left(\frac{d^2y}{dx^2} \right) + 3x^2 \left(\frac{dy}{dx} \right) + (3 - 6x)x^{2y} = x^4 + 2x - 5$
- 3) Show that the equation $(y^2 + 2x^2) \frac{d^2y}{dx^2} + 2(y + x) \left(\frac{dy}{dx} \right)^2 + x \frac{dy}{dx} + y = 0$ is exact and find its first integral.
- 4) Solve
 - a) $\frac{d^3y}{dx^3} = xe^x$
 - b) $\frac{d^2y}{dx^2} = \sec y^2 \tan y$
- 5) Solve
 - a) $y^3 y'' = a$
 - b) $x^2 \frac{d^4y}{dx^4} + 1 = 0$.

Practical No.6: Linear Differential Equations of second order

- 1) Find the general solution of $(1 - x^2)y'' - 2xy' + 2y = 0$
if $y=x$ is a solution of it.
- 2) Solve $x^2 y'' + xy' - y = 0$, give that $x + \frac{1}{x}$ is one integral.
- 3) Solve $x^2 y'' - (x^2 + 2x)y' + (x + 2)y = x^3 e^x$.
- 4) Solve $y'' - 2 \tan y y' + 5y = x^3 e^x \sec x$.
- 5) Solve by change of dependence variable
 - a) $y'' - 4xy' + (4x^2 - 1)y = -3e^{x^2} \sin x$.
 - b) $y'' - \left(\frac{2}{x}\right)y' + \left(1 + \frac{2}{x^2}\right)y = xe^x$.

Practical No.7: Power series method

- 1) Determine whether $x=0$ is an ordinary point or regular singular point for the differential equations
 - a) $2x^2y'' - xy' + (x - 5)y = 0$.
 - b) $(x^2 + 1)y'' + xy' - xy = 0$.
- 2) Show that $x=0$ is a regular singular point and $x=1$ is an irregular singular point of $x(x - 1)^3y'' + 2(x - 1)^3y' + 3y = 0$.
- 3) Find series solution of $y'' - xy' + x^2y = 0$ about $x=0$.
- 4) Solve in series: $9x(1-x)y'' - 12y' + 4y = 0$.
- 5) Solve $2x^2y'' - xy' + (1 - x^2)y = 0$.
- 6) Find solution near $x=0$ of $x^2y'' + (x + x^2)y' + (x - 9)y = 0$.

Practical No.8: Linear partial differential equations of the first order

- 1) Form a partial differential equation by eliminating the arbitrary function ϕ from $\phi(x^2 + y^2 + z^2, z^2 - 2xy) = 0$, what is the order of this partial differential?
- 2) Eliminate the arbitrary function f and g and obtain the partial differential equation from $z = f(x^2 - y) + g(x^2 + y)$.
- 3) If u is a function of x, y and z which satisfies the partial differential equation $(y - z)\frac{\partial u}{\partial x} + (z - x)\frac{\partial u}{\partial y} + (x + y)\frac{\partial u}{\partial z} = 0$ show that u contain x, y and z only in combination of $x + y + z$ and $x^2 + y^2 + z^2$.
- 4) Find the general integral of
 - a) $xzp + yzq = xy$.
 - b) $(y + zx)p - (x + yz)q = x^2 - y^2$.
- 5) Find the integral surface of the partial differential equation $(x - y)y^2p + (y - x)x^2q = (x^2 + y^2)z$ Passing through the curve $xz = a^3, y = 0$
- 6) Find the surface which intersect the surfaces of the system $z(x + y) = c(3z + 1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$.

T.Y.B.Sc Mathematical Practical Course MTH-369
Based on MTH-365(A) OR MTH-365(B) , MTH-366(A) OR
366(B)

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Practical No. 1(A): Linear programming problem

- 1) A farmer has a 100 acre farm. He can sell the tomatoes, lettuce, or radishes he can raise. The price he can obtain is Rs. 1 per kilogram for tomatoes, Rs. 0.75 head for lettuce and Rs. 2 per kilogram for radishes. The average yield per acre is 2,000 kilograms of tomatoes, 3,000 heads of lettuce and 1,000 kilograms of radishes. Fertilizers is available at Rs. 0.50 per kilogram and the amount required per acre is 100 kilograms each for tomatoes and lettuce and 50 kilograms for radishes. Labour required for sowing, cultivation and harvesting per acre is 5 man/days for tomatoes and radishes and 6 man/days for lettuce. A total of 400 men/days of labour are available at Rs. 20/- per man/days.

Formulate this problem as a L. P. P. to maximize the farmer's total profit.

- 2) Solve the problem graphically

$$\text{Maximize } Z = 4x_1 + 3x_2$$

Subject to constraints

$$2x_1 + x_2 \leq 72, \quad x_1 + 2x_2 \leq 48$$

$$x_1 \geq 0, \quad x_2 \geq 0$$

- 3) Solve the following L.P.P. by Big-M method :

$$\text{Maximize } z = 5x_1 + 6x_2 \quad \text{subject to}$$

$$2x_1 + 5x_2 \geq 1500, \quad 3x_1 + x_2 \geq 1200$$

$$x_1, x_2 \geq 0.$$

- 4) Solve the following L.P.P. by simplex method. Find alternate solution if exists :

$$\text{Maximize } z = 4x_1 + 10x_2 \quad \text{subject to}$$

$$2x_1 + x_2 \leq 50, \quad 2x_1 + 5x_2 \leq 100, \quad 2x_1 + 3x_2 \leq 90$$

$$x_1, x_2 \geq 0.$$

- 5) Solve the following L.P.P.

$$\text{Maximize } z = 5x_1 + 3x_2 \quad \text{subject to}$$

$$x_1 + x_2 \leq 2, \quad 5x_1 + 2x_2 \leq 10, \quad 3x_1 + 8x_2 \leq 12$$

$$x_1, x_2 \geq 0$$

Is there a degenerate solution for given L. P. P.? Resolve it.

Practical No 2(A) : Transportation Problem (TP)

1. Find the initial basic feasible solution of the following transportation problem by
(a) North-West Corner Rule (b) Matrix Minima Method.

Warehouse→ Factory ↓	W ₁	W ₂	W ₃	W ₄	Factory Capacity
F ₁	19	30	50	10	7
F ₂	70	30	40	60	9
F ₃	40	8	70	20	18
Warehouse Requirement	5	8	7	14	34

2. Determine an initial basic feasible solution of the following TP by using Vogel's Approximation Method.

		Destination				
		1	2	3	4	
Source	I	21	16	15	13	11
	II	17	18	14	23	13
	III	32	27	18	41	19
Demand		6	10	12	15	43

3. Determine the optimal basic feasible solution to the following TP.

	D ₁	D ₂	D ₃	D ₄	Available
O ₁	1	2	1	4	30
O ₂	3	3	2	1	50
O ₃	4	2	5	9	20
Required	20	40	30	10	100

Is there exists alternative optimal solution?

4. A product is produced by four factories A, B, C and D. The unit production costs in them are Rs 2, Rs 3, Rs 1 and Rs 5 respectively. Their production capacities are: factory A- 50 units, B-70 units, C-30 units and D-50 units. These factories supply the product to four stores, demands of which are 25, 35, 105 and 20 units respectively. Unit transport cost in rupees from each factory to each store is given in the table as:

		Stores			
		1	2	3	4
Factories	A	2	4	6	11
	B	10	8	7	5
	C	13	3	9	12
	D	4	6	8	3

Determine the extent of deliveries from each of the factories to each of the stores so that the total production and transportation cost is minimum.

5. Solve the following transportation problem

	A	B	C	D	E	Capacity
1	13	---	31	8	20	200
2	14	9	17	6	10	175
3	25	11	12	17	15	150
4	10	21	13	---	17	325
Demand	110	90	120	230	160	

Practical No.3(A) : Assignment Problem (AP)

1. Solve the following assignment problem

	I	II	III	IV	V
1	11	17	8	16	20
2	9	7	12	6	15
3	13	16	15	12	16
4	21	24	17	28	26
5	14	10	12	11	13

2. A company has one surplus truck in each of the cities A, B, C, D and E and one deficit truck in each of the cities 1, 2, 3, 4, 5 and 6. The distance between the cities in kms is shown in the table. Find the assignment of trucks from cities in surplus to cities in deficit so that total distance covered by vehicles is minimum.

	1	2	3	4	5	6
A	12	10	15	22	18	8
B	10	18	25	15	16	12
C	11	10	3	8	5	9
D	6	14	10	13	13	12
E	8	12	11	7	13	10

3. Find the optimal assignment schedule of following AP.

	A	B	C	D	E
1	4	6	10	5	6
2	7	4	---	5	4
3	---	6	9	6	2
4	9	3	7	2	3

4. Solve following AP.

	I	II	III	IV
A	2	3	4	5
B	4	5	6	7
C	7	8	9	8
D	3	5	8	4

Is there exist alternative solution?

5. A team of 5 horses and 5 riders has entered a jumping show contest. The number of penalty points to be expected when each rider rides any horse is shown below:

	R1	R2	R3	R4	R5
H1	5	3	4	7	1
H2	2	3	7	6	5
H3	4	1	5	2	4
H4	6	8	1	2	3
H5	4	2	5	7	1

How should the horses be allotted to the riders so as to minimize the expected loss of the team?

Practical No. 4(A) : Game Theory

1) The pay off matrix of a game is given below. Find the solution of the game.

Player A	Player B				
	I	II	III	IV	V
I	-2	0	0	5	3
II	3	2	1	2	2
III	-4	-3	0	-2	6
IV	5	3	-4	2	-6

2) Solve the game by principle of dominance:

Player A	Player B			
	I	II	III	IV
I	3	2	4	0
II	3	4	2	4
III	4	2	4	0
IV	0	4	0	8

3) In a game of matching coins with two players, suppose A wins one unit of value when there are two heads, wins nothing when there are two tails and loses $\frac{1}{2}$ unit of value when there are one head and one tail. Determine the pay-off matrix, the best strategies for each player, and the value of the game to A.

4) Solve the following game graphically :

Player A	Player B		
	I	II	III
I	-2	0	-8
II	-4	-5	-3

5) Solve the following game graphically :

		Player B	
Player A		I	II
	I	2	7
	II	3	5
	III	11	2

MTH:365(B) Dynamics

Practical No. 1(B) kinematics

- The position of a moving point at a time t is given by $x = a \cos t$ and $y = a \sin t$, Find its paths, velocity and acceleration.
- A particle is constrained to move along the equiangular spiral $r = e^{b\theta}$ so that the radius vector moves with constant angular velocity ω , Determine the radial and transverse components of velocity and acceleration.
- A small bead slides with constant speed V on a smooth wire in the shape of the cardioid $r = a(1 + \cos \theta)$. Show that the angular velocity is $\left(\frac{V}{2a}\right) \sec\left(\frac{\theta}{2}\right)$ and that the radial components of acceleration are constant.
- Prove that the acceleration of a point moving in a curve with uniform speed is $\rho\psi^2$. A point describes a cycloid $S = 4a \sin \psi$ with uniform speed V . Find its acceleration at any points.
- A point moves in a plane curve so that its longitudinal acceleration is constant and the magnitude of the tangential velocity and the normal acceleration are in a constant ratio. Show that the intrinsic equation of the paths of the form $S = A\psi^2 + B\psi + C$.
- A particle describes a curve (for which s and ψ with simultaneously) with uniform speed V . If the retardation at any point S be $\frac{V^2 c}{s^2 + c^2}$. Prove that curve is a catenary.

Practical No. 2 (B) Rectilinear Motion

- Two cars start off to race with velocities u and v and travel with straight line with uniform acceleration α and β , if the race ends in a dead heat, prove that the length of the course is $\frac{2(u-v)(u\beta - v\alpha)}{(\alpha - \beta)^2}$.
- A body travels a distance S in t seconds. It starts from rest and ends at rest. In the first part of the journey it moves with constant acceleration f and in the second part with constant retardation r . Show that $t = \sqrt{2s \left(\frac{1}{f} + \frac{1}{r}\right)}$.
- In a S.H.M. of amplitude a and period T , prove that $\int_0^T V^2 dt = \frac{2\pi^2 a^2}{T}$.
- A point in straight line with S.H.M. has velocities v_1 and v_2 when its distances from the centre are x_1 and x_2 . Show that the period of motion is $2\pi \sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$.
- A particle m is attached to a light wire which is stretched tightly between two fixed points with a tension T . If a, b are the distances of the particles from the two ends. Prove that the period of a small transverse oscillation of mass is $2\pi \sqrt{\frac{mab}{T(a+b)}}$.

6. A mass m hangs from a fixed point by a light spring and is given by a small vertical oscillation. If l is the length of the spring in equilibrium position and n is number of the oscillation per second. Show that the natural length of the spring is $(l - \frac{g}{4\pi^2 n^2})$.

Practical No. 3(B) Uniplanar Motion

1. A particle is thrown over a triangle from one end of a horizontal base and grazing over the vertex falls on the other end of the base. If A, B be the base angles of the triangle and α the angle of projection. Prove that $\tan \alpha = \tan A + \tan B$.
2. If t be the time in which projectile reaches a point P in its path and t' the time from P till it reaches the horizontal plane through the point of projection. Show that the height of P above a horizontal plane is $\frac{1}{2}gtt'$.
3. Two particles are projected from the same point in the same vertical line with equal velocities. If t, t' be the times taken to reach the other common point of their path and T, T' the times to the highest point. Show that $tT + t'T'$ is independent of the direction of projection.
4. A body projected at an angle α to the horizontal so as to clear two walls of equal height a at a distant $2a$ from each other. Show that the range is equal to $2a \cot \frac{\alpha}{2}$.
5. A ball is projected so as to just clear two walls, first of height a at a distant b from the point of projection and the second of height b and at a distant a from the point of projection, show the range on the horizontal plane is $\frac{a^2 + ab + b^2}{a + b}$.
6. A particle is projected under gravity with velocity $\sqrt{2ga}$ from a point at height h above a low level plane. Show that the angle of projection θ for the maximum range on the plane is given by $\tan \theta^2 = \frac{a}{a+h}$ and the maximum range is $2\sqrt{a(a+h)}$.

Practical No. 4(B) Central orbits

1. If orbit is an equiangular spiral $r = e^{\theta \cot \alpha}$ under a force P to the pole, Prove that the law of force $P = \frac{\mu}{r^3}$.
2. A particle describes the curve $r^2 = a^2 \sin 2\theta$ under a force toward the pole. Find the law of force.
3. If orbit is $r^n \cos n\theta = a^n$ then prove that $P = \mu r^{2n-3}$.
4. A particle describes the curve $p^2 = ar$ under a central force to the pole. Find the law of force.
5. If orbit is a cardioid $r = a(1 + \cos \theta)$ under a central force to the pole, then prove that the law of force $P = \frac{\mu}{r^4}$.
6. A particle moves with a central acceleration which varies inversely as cube of the distance, if it be projected from an apse at a distance a from the origin with velocity

which is $\sqrt{2}$ times the velocity from a circle of radius a , then show that equation to the path is $r \cos \frac{\theta}{\sqrt{2}} = a$.

MTH-366(A): Applied Numerical Methods

Practical No 5 (A) Simultaneous Linear Algebraic Equations.

1. Solve following systems of linear equations by Factorization (or Triangularisation) method.

$$\begin{aligned} \text{i)} \quad & 10x + y + z = 12 \\ & 2x + 10y + z = 13 \\ & x + y + 5z = 7 \end{aligned}$$

$$\begin{aligned} \text{ii)} \quad & 3x + y + 2z = 16 \\ & 2x - 6y + 8z = 24 \\ & 5x + 4y - 3z = 2 \end{aligned}$$

2. Solve the following system of linear equations by Crout's Method.

$$\begin{aligned} 3x + y + 2z &= 3 \\ 2x - 3y - z &= -3 \\ x - 2y + z &= -4 \end{aligned}$$

3. Find the inverse of the following matrix by Crout's method

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

4. Solve the following system of linear equations by Gauss Seidel iteration method.

$$\begin{aligned} 2x + y + z &= 4 \\ x + 2y + z &= 4 \\ x + y + 2z &= 4 \end{aligned}$$

5. Solve the following system of linear equations by Relaxation method.

$$\begin{aligned} 10x + y + z &= 12 \\ 2x + 10y + z &= 13 \\ 2x + 2y + 10z &= 14 \end{aligned}$$

Practical 6(A) Interpolation

1. Find Newton's interpolation polynomial satisfied by the following table.

x	-4	-1	0	2	5
f(x)	1245	33	5	9	1335

2. The following table gives some relation between steam pressure & temperature.

Using Newton's divided difference formula find the pressure at 372.1°

Temp $^\circ$ C	361°	367°	378°	387°	399°
Pressure (kgf/cm 2)	154.9	167.9	191	212.5	244.2

3. Solve the equation

$$x^3 - 6x - 11 = 0 \quad (3 < x < 4) \quad \text{by iterative method. Take } h=0.2$$

(Hint: find x when y = 0, taking h=0.2)

4. From the following table

x	1.8	2.0	2.2	2.4	2.6
y	2.9	3.6	4.4	5.5	6.7

find x when y=5, using iterative method.

5. Find whether following function is cubic spline or not.

$$f(x) = \begin{cases} 5x^3 - 3x^2 & \text{if } -1 \leq x \leq 0 \\ -5x^3 - 3x^2 & \text{if } 0 \leq x \leq 1 \end{cases}$$

6. Obtain cubic spline approximation for the following dataD

x	0	1	2
f(x)	-1	3	29

with $M_0 = 0, M_2 = 0$ Hence interpolate at $x=0.5$ & 1.5

Practical 7(A) : Numerical Differentiation & Integration.

1. Find the first & second derivatives of the function tabulated below at the point $x=1.9$

x	1.0	1.2	1.4	1.6	1.8	2.0
f(x)	0	0.128	0.544	1.296	2.432	4.00

2. The following data gives corresponding values of pressure & specific volume of super-heated steam.

v	2	4	6	8	10
p	105	42.07	25.3	16.7	13

Find the rate of change of volume with respect to pressure when $v = 2$.

3. Evaluate the integral $\int_1^2 \int_1^2 \frac{dx dy}{x+y}$ using the trapezium rule with $h=k= 0.25$

4. Evaluate the integral $\int_0^1 \int_0^1 e^{x+y} dx dy$ using the trapezium rule & Simpson rule with $h=k= 0.5$.

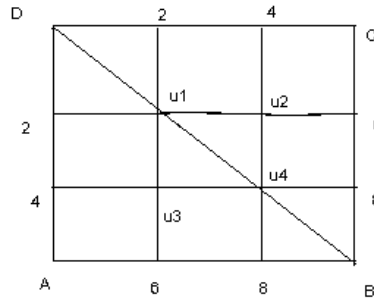
Obtain the exact solution & find the magnitude of errors in two solutions.

5. Evaluate the integral $\int_1^{1.5} \int_1^{1.5} \frac{dx dy}{(x^2+y^2)^{1/2}}$ using the trapezium rule &

Simpson rule with two subintervals. Take $h=k= 0.25$.

Practical 8(A) Boundary value problems in ordinary differential equations and initial and boundary value problems in partial differential equations.

- Solve the boundary value problem $x y'' + y = 0$, $y(1)=1$, $y(2)=2$ by second order finite difference method with $h = 0.25$
- Using second order finite difference method, find $y(0.25)$, $y(0.5)$, $y(0.75)$ satisfying the differential equation $y'' - y = x$ and subject to the condition $y(0) = 0$, $y(1) = 2$
- Classify the following partial differential equations.
 - $2u_{xx} + 3u_{yy} - u_x + 2u_y = 0$
 - $u_{xx} + 2x u_{xy} + (1 - y^2) u_{yy} = 0$
- Solve $u_{xx} + u_{yy} = 0$ numerically for the following mesh with uniform spacing & with boundary conditions as shown below in the figure.



5. Solve the boundary value problem

$u_{xx} + u_{yy} = x + y + 1$, $0 \leq x \leq 1$, $0 \leq y \leq 1$ and $u = 0$ on the boundary, numerically using five point formula & Liebmann iteration, with mesh length $h = 1/3$.

MTH-366(B): Differential Geometry

Practical No. 5(B) Curves in Spaces

- 1) Find the equation of the osculating plane at a general point u on the helix $x = a \cos u$, $y = a \sin u$, $z = cu$
- 2) If the tangent and binormal at a point of a curve makes angles α and β respectively with fixed direction, show that $\frac{d\alpha}{d\beta} = \frac{\sigma \sin \beta}{\rho \sin \alpha}$.
- 3) Show that Serret-Frenet formulae can be written in the form as $\omega \times t = t'$, $\omega \times n = n'$, $\omega \times b = b'$
- 4) Find the curvature and torsion of the cubic curve $\vec{r} = (u, u^2, u^3)$.
- 5) For any curve prove that $[b', b'' b'''] = \tau^3 \tau^3 (\kappa' \tau - \tau' \kappa) = \tau^5 \frac{d}{ds} \left(\frac{\kappa}{\tau} \right)$
- 6) Show that for an helix $\frac{\kappa}{\tau} = \text{constant}$.
- 7) For the curve $x = 3u$, $y = 3u^2$, $z = 2y^3$. Show that curvature and torsion are equal.

Practical No. 6(B) Curvatures

- 1) If R is the radius of spherical curvature then show that $R = \left| \frac{t' \times t''}{\kappa^2 \tau} \right|^2$.
- 2) Show that a curve lies on a sphere then prove that $\frac{d}{ds} (\sigma \rho^2) = -\frac{\rho}{\sigma}$
- 3) Show that the radius of spherical curvature of a circular helix $x = a \cos \theta$, $y = a \sin \theta$, $z = a\theta \cot \alpha$ is equal to the radius of circular curvature.
- 4) In any curve if R is the radius of spherical curvature and ρ is the radius of curvature and σ is the radius of torsion at any point. (x, y, z) then Show that

$$(x''')^2 + (y''')^2 + (z''')^2 = \rho^4 \left[1 + \frac{R^2}{\sigma^2} \right].$$

- 5) Find the equation of involutes of the circular helix $x = a \cos \theta$, $y = a \sin \theta$, $z = a\theta \tan \alpha$
- 6) Prove that the distance between corresponding points of two Bertrand curve is constant.
- 7) Investigate the spherical indicatrix (image) of the curve $x = a \cos u$, $y = a \sin u$, $z = cu$, $c \neq 0$

Practical No. 7(B) Envelopes of Surfaces

- 1) Find the equation of tangent plane and normal to the surface $xy + yz + zx = xyz$ at $(1,1,1)$.
- 2) Find the envelope of the plane $\frac{x}{a} \cos \theta \sin \phi + \frac{y}{b} \sin \theta \sin \phi + \frac{z}{c} \cos \phi = 1$
- 3) Show that the envelope to the family of Paraboloids $x^2 + y^2 = 4a(z - a)$ is the circular cone $x^2 + y^2 = z^2$
- 4) Calculate the first and second fundamental magnitudes for the paraboloid $\bar{r} = (u, v, u^2 - v^2)$
- 5) Prove that the metric is a positive definite form in du and dv .
- 6) Find the envelopes of the plane $3xt^2 - 3yt + z = t^3$ and show that its edge of regression is the curve of intersection of the surface $y^2 = zx, xy = z$.
- 7) Find the Equation of tangent and normal to the given surface $x^3 + y^3 + z^3 - 3xyz = 5$ at $(1,2,3)$

Practical No. 8(B) Developable Surfaces

- 1) Show that surface $xy = (z-c)^2$ is developable
- 2) Prove that $z = y \sin x$ is ruled surface.
- 3) Find equations to the developable surface which has helix $x = a \cos u, y = a \sin u, z = cu$ for its edge of regression.
- 4) Prove that the locus of the center of curvature is an evolute only when the curve is plane.
- 5) Show that the surface $z = f(x, y)$ should represent a developable surface if and only if $rt = s^2$ where $r = \frac{\partial^2 f}{\partial x^2}, s = \frac{\partial^2 f}{\partial x \partial y}, t = \frac{\partial^2 f}{\partial y^2}$
- 6) Are the following curves developable? Justify
 - a) $xyz = 4$.
 - b) $z = xy$