NORTH MAHARASHTRA UNIVERSITY, JALGAON

SYLLABUS FOR SECOND YEAR B.Sc. (S.Y.B.Sc.) STATISTICS (PRINCIPAL)
FROM JUNE 1993

Paper I: Distribution Theory

1. Univariate Discrete & Continuous Probability Distributions.

1.1 Definition of countably infinite sample space with illustrations.

1.2 Random variable (r.v.) Definition-Discrete random variable, continuous random variable, probability mass function (p.m.f.) probability density function (p.d.f.), Distribution function, properties of distribution function.

1.3 Expectation of a r.v., moments, relation between raw and central moments (up to fourth order).

1.4 Definition of bivariate continuous probability distribution, marginal and contitional distributions, independence of two r.v.s., Theorems on expectation.

$E(ax \pm by) = aE(X) \pm bE(Y)$

$E(ax \cdot by) = abE(X)\ E(Y)$ for X,Y independent r.v.s.

1.5 Moment generating function (MGF), properties of MGF

i) Effect of change of origin & Scale.

ii) Statement of uniqueness property.

iii) MGF of sum of two independent r.v.s.

1.6 Cumulant generating function (CGF): Definition of CGF & relations of cumulants with moments. (up to forth order only). Properties of CGF:

(i) Effect of change of origin & scale (ii) Additive property of cumulants.

1.7 Mode, Median, Quartiles and Mean deviation for continuous r.v.

1.8 p.d.f. of simple functions of r.v.s. such as $Y = AX+B$, $Y=X^2$, $Y=\log X$., $Y= e^X$

1.9 Examples & Problems.

2. Some Standard discrete distributions:

Poisson distribution

2.1 pmf : $P(x=x) = e^{-m} \frac{m^x}{X!}$, $x=0,1,2 \ldots$, $m > 0$.

2.2 Mean, Variance, MGF, CGF.

2.3 Additive property & its extension to n independent, Poisson r.v.s.

2.4 Recurrence relation for probabilities.

2.5 Recurrence relations for raw & central moments.

2.6 Mode.

2.7 Conditional distribution of X given (X+Y),

2.8 Poisson distribution as a limiting form of the binomial distribution.

2.9 Illustrations of real life situations where Poisson distribution is applicable.

2.10 Examples and problems.

3. Geometric Distribution:

3.1 pmf: $P(X=X) = q^X p$, $X=0, 1, 2, \ldots$, $p+q=1$, $p, q \geq 0$, Distribution function.

3.2 Moments, MGF, CGF.

3.3 Geometric distribution as a waiting time distribution, Lack of memory properity.

3.4 Examples and Problems.

4. Some Standard Continuous Univariate distributions.
Normal distribution:

4.1. pdf

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad -\infty < \mu < \infty,\ \sigma > 0,\ -\infty < x < \infty$$

Identification of the parameters μ & σ^2, Nature of probability curve.

4.2 Properties of the distribution, Distribution function.

4.3 Mean, Variance, Moments.

4.4 MGF, CGF, cumulants, β_1, β_2, γ_1, γ_2

4.5 Recurrence relation for central moments.

4.6 Additive property.

4.7 Computation of probabilities using normal probability tables.

4.8 Normal approximation of binomial & Poisson distribution.

4.9 Standard normal distribution.

4.10 Examples and problems.

5. Exponetial distribution:

5.1 pdf $F(x) = \frac{1}{\theta} e^{-x/\theta} \quad x \geq 0, \quad \theta > 0$

Identification of parameters, Nature of probability curve.

5.2 Moments, mgf, cgf, distribution function, median, quartiles

5.3 Lack of memory property.

5.4 Examples and Problems.

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6. Gama distribution:

6.1 p.d.f. of Gamma distribution (Two parameter form)

$F(x) = \frac{\alpha^{\lambda}}{\Gamma\lambda} e^{-\alpha x} \, x^{\lambda-1} \quad 0 \leq x < \infty$

Notation: $X \sim G(\alpha, \lambda)$

6.2 Special cases: $0 < \alpha, \lambda < \infty$

i) α =1,

ii) λ =1 (Exponential distribution)

6.3 Mean, Variance MGF, CGF, moments, cumulants, mode, γ_1 and γ_2

6.4 Additive property.

6.5 Distribution of the sum of i. i d. exponential variates.

6.6 Relation between distribution functions of Poisson & Gamma variates.

6.7 Recurrence relation for central moments.

6.8 Examples and Problems.

7. Beta distributions of first & second kind.

7.1 p.d.f. of Beta distribution of first kind

$F(x) = \frac{x^{m-1}(1-x)^{n-1}}{\beta(m,n)} \quad 0 \leq x \leq 1, \quad m, n > 0$

Notation: $X \sim \beta_1(m,n)$

p.d.f. of Beta distribution of second kind

$F(x) = \frac{x^{m-1}}{\beta(m,n)(1+x)^{m+n}} \quad 0 \leq x < \infty, \quad m, n > 0$

Notation: $X \sim \beta_2(m,n)$

7.2 Relation between two kind of variates.

7.3 Mean, variance, mode. The r th raw moment.

7.4 Distributions of X/Y and X/(X+Y) where X and Y are independent Gamma variates.

7.5 Examples and Problems.

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8. Chi-square distribution

8.1 Definition of chi-square variate as sum of squares of n i.i.d. standard normal variates.

8.2 Derivation of p.d.-. of chi-square with n degrees of freedom (using MGF)

8.3 Nature of probability curve.

8.4 Use of chi-square tables for calculation of probabilities.

8.5 Mean, variance, mode mgf, cgf, moments, γ_1, γ_2

8.6 Additive property.

8.7 Distributions of

$$\frac{x_1^2}{x_1^2 + x_2^2} \quad \text{and} \quad \frac{x_1^2}{x_2^2}$$ where x_1^2 & x_2^2 are independent chi-square variates.

8.8 Examples and Problems.

9. t-distribution:

9.1 Definition of t-statistic with n d.f. in the form of $t = \frac{u}{\sqrt{\chi_n^2/n}}$ where u is N(o,1), χ_n^2 is a chi-square variate with n d.f. & u and χ_n^2 are independent variates.

9.2 Derivation of p.d.f.

9.3 Nature of probability curve.

9.4 Mean, variance & moments.

9.5 Statement of Normal approximation to t distribution.

9.6 Use of probability tables for calculation of probabilities.

9.7 Examples and Problems.

10. F distribution:

10.1 Definition of F with n1 and n2 degree of freedom as $F_{n_1, n_2} = \frac{\chi_1^2/n_1}{\chi_2^2/n_2}$ where χ_1^2 & χ_2^2 are independent chi-square variate with n1 & n2 d.f. respectively.

10.2 Derivation of p.d.f.

10.3 Nature of probability curve

10.4 Mean, variance, moments, mode.

10.5 Interrelations among normal, chi-square, t & F variates

10.6 Use of F tables for calculation of probabilities.

10.7 Examples and problems.

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PAPER II : MATHEMATICAL STATISTICS

1. Gamma & Beta functions.

1.1 Definition of gamma function

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1}\, dx \quad n > 0$$

Recurrence Relation (Derivation)

$$\Gamma(n+1) = n\,\Gamma(n)$$

Evaluation of Integrals related to gamma function such as

i) $\int_0^\infty e^{-ax} x^{n-1}\, dx$ ii) $\int_0^\infty e^{-ax^2} x^{b-1}\, dx$

1.2 Definition of Beta function

(i) $B(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1}\, dx \quad m > 0 \quad n > 0$

(ii) $B(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\, dx$

Properties.

$B(m,n) = B(n,m)$ ...

$B(1/2, 1/2) = \pi$

1.3 Relation between gamma & beta functions

$$B(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$$

1.4 Examples & Problems.

2. Double Integrals:

2.1 Evaluation of a double integral with repeated integral (statement only). Evaluation of simple double integrals over regions bounded by straight lines.

2.2 Transformation of variables including rectangular to polar, Transformation of regin of integration, Jacobian of the transformation, Evaluation of double integrals using transformations.

2.3 Examples and Problems.

3. Multiple Regression (trivariate case-sample data only)

3.1 Yule's notation.

3.2 Fitting of Regression planes by the method of least squares. Definition of partial regression coefficients bij.k, and its interpretation.

3.3 Residuals: Definition, Order, Derivation of variances & Co-variances.

3.4 Examples and Problems.

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4. Multiple & Partial correlations (trivariate case.sample data only)

4.1 Definitbn of multiple correlation coefficient Ri-jk as the correlation coefficient between a variable & its best linear estimater.

4.2 Derivation of the formula for the multiple correlation coefficient in terms of co-factors of the correlation matrix, and simple correlation coefficients.

4.3 Properties of multiple correlation coefficient

(a) $0 \leqslant$ Ri.jk $\leqslant 1$ (b) Ri.jk $\geqslant$ rij

4.4 Interpretation of

(a) Ri. jk = 1

(b) Ri. jk = 0

4.5 Definition of partial correlation coefficient rij.k as correlation coefficient between residuals.

4.6 Derivation of the formula for rij.k. in terms of the cofactors of the correlation matrix and simple correlation coefficients.

4.7 Properties of partial correlation coefficient:

(a) $-1 \leqslant$ rij.k $\leqslant 1$

(b) b.ji.k. bij.k. = r^2ij.k.

4.8 Examples and problems.

5. Sampling distributions.

5.1 Random sample from a continuas distribution as i.i.d.r.v.s. $X_1, X_2 \ldots\ldots X_n$

5.2 Notion of statistics as a function $T(X_1, X_2 \ldots\ldots X_n)$ with illistrations.

5.3 Sampling distribution of $T(X_1, X_2 \ldots\ldots X_n)$ Distribution of sample mean x from normal, exponentiol & Gamma distributions, Notion of a standard error of a statistic.

5.4 Examples & Problems.

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6. Tests

6.1 Notion of hypothesis, null hypothesis, alternative hypothesis Critical region, Level of significance. Test of significance, one & two tail tests.

7. Large sample Tests (Two tail tests)

7.1 $H_o : \mu = \mu_0$ $\quad H_1 : \mu \neq \mu_0$

7.2 $H_o : \mu_1 = \mu_2$ $\quad H_1 : \mu_1 \neq \mu_2$

7.3 $H_o : P = P_o$, $\quad H_1 : P \neq P_o$

7.4 $H_o : P_1 = P_2$, $\quad H_1 : P_1 \neq P_2$

7.5 Examples and problems.

8. Small sample tests:

8.1 Tests based on chi-square distribution

(i) Test for independance of attiributes (Yate's correction not expected) (ii) Test of goodness of fit.

(iii) Tests for $\sigma^2 = \sigma_0^2$ for μ known & unknown

8.2 Tests based on t-distribution.

i. $H_o : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$

ii. $H_o : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$ & paired t-test

iii. $H_o : \rho = 0$ against $H_1 : \rho \neq 0$

8.3 F test for

$H_o : \sigma_1^2 = \sigma_2^2$

8.4 Examples and Problems.

9. Interpolation:

9.1 The operator Δ & E. Their simple properties and inter-relations.
Construction of difference table. Fundamental theorem of finite differences: If f(x) is a polynomial of n^{th} degree them,

$\Delta^n f(x) = \text{constant}.$

9.2 Interpolation for equal and unequal intervals, Newton's forward difference formula and Lagrange's formula(without proof).

9.3 Examples and problems.

10. Numerical Integration:

10.1 Derivation of General quadrature formula. Derivation of trapezoidal rule.

10.2 Simpson's 1/3rd and 3/8th rules as particular cases of general quadrature formula.

10.3 Examples and Problems.

List of books recommended:

1. Introduction to mathematical statistics by R.V.Hogg & A.J.Craig.
2. Introduction to theory of statistics by Mood Graybill & Boes.
3. Mathematical Statistics by S.C.Gupta & V.K.Kapoor.
4. Mathematical Statistics by B.D.Gupta & O.P.Gupta
5. Calculus of finite differences by H.C.Saxena
6. Calculus of finite differences & numerical analysis.

Paper III PRACTICALS:-

Note: 1. Students must complete all the practicals in each practical paper to the satisfaction of teacher concerned.

2. Students must produce at the time of the practical examination the laboratory journal alongwith the completion certificate signed by the Head of the Department.

3. Of the 100 marks for each Practical examination, 10 marks shall be reserved for viva-voca and 10 marks for Journal. Thus the Practical paper shall actually carry 80 marks.

4. The duration of praccical exam. will be 3 hours.

Title of the Experiment :-

1. Multiple Regression: Fitting regression planes for trivariate data given the sample means, variances and correlation matrix. (computations of means, variances,correlations from raw data are not expected)

2. Fitting of regression planes for trivariate data given the sum, sum of squares and sum of products.

3. Computation of multiple and partial correlations from the sample correlation matrix.

4. Fitting of Poisson Distribution (Calculations of expected frequencies are expected)

5. Applications of Poisson and geometric distribution for calculation of probabilities.

6. Model sampling from Poisson Distribution.

7. Fitting of normal distribution & calculation of expected frequencies.

8. Model sampling from (a) exponential and (b) normal distribution. p.d.f.
$F(x) = \frac{1}{\theta} e^{-x/\theta} \quad x \geq 0 \quad \theta > 0$
where parameter is explicitly stated.

9. Applications of normal distribution and exponential distribution for calculation of probabilities.

10. Large sample tests:
    (a) $P = P_0$ against $P \neq P_0$
    (b) $P_1 = P_2$ against $P_1 \neq P_2$
    (c) $\mu = \mu_0$ against $\mu \neq \mu_0$
    (d) $\mu_1 = \mu_2$ against $\mu_1 \neq \mu_2$

11. Test based on t distribution:
    (a) $\mu = \mu_0$ against $\mu \neq \mu_0$
    (b) $\mu_1 = \mu_2$ against $\mu_1 \neq \mu_2$
    (c) $\rho = 0$ against $\rho \neq 0$

12. Tests based on Chi-square distribution:
    (a) $\sigma^2 = \sigma_0^2$ for μ known & unknown.
    (b) independence of attributes.
    (c) test of goodness of fit.

13. Test based on F distribution.
$\sigma_1^2 = \sigma_2^2$

14. Interpolation for equal & unequal intervals.

15. Numerical integration (Trapezoidal, Simpson's $\frac{1}{3}$ rule, Simpson's $\frac{3}{8}$ rule)

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