NORTH MAHARASHTRA UNIVERSITY JALGAON.

Syllabus for S.Y.B.Sc. (Mathematics)

With effect from June 2016. (Semester system).

The pattern of examination of theory and practical papers is semester system. Each paper is of 100 marks (60 marks external and 40 marks internal). and practical course is of 100 marks (60 marks external and 40 marks internal). The examination will be conducted at the end of each semester.

STRUCTURE OF COURSES

SEMESTER –I

MTH-231 : Calculus of Several Variables

MTH-232 (A) : Algebra
OR
MTH-232 (B) : Theory of Groups

MTH- 233 : Practical Course based on MTH-231, MTH-232

SEMESTER- II.

MTH-241 : Complex Variables

MTH-242 (A) : Differential Equations
OR
MTH-242(B) : Differential and Difference Equations

MTH-243 : Practical Course based on MTH-241, MTH-242

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SEMESTER – I

MTH -231: Calculus of Several Variables

Unit- 1 : Functions of Two and Three variables **Periods -15 ,Marks – 15**

1.1 Limits and Continuity
1.2 Partial Derivatives and Jacobians .
1.3 Higher order Partial Derivatives
1.4 Differentiability and Differentials
1.5 Necessary and sufficient conditions for Differentiability
1.6 Schwarz's Theorem
1.7 Young's Theorem.

Unit- 2 : Composite Functions and Mean Value Theorem **Periods – 15, Marks – 15**

2.1 Composite functions. Chain Rule.
2.2 Homogeneous functions.
2.3 Euler's Theorem on Homogeneous Functions.
2.4 Mean Value Theorem for Function of Two Variables.

Unit -3 : Taylor's Theorem and Extreme Values **Periods – 15, Marks – 15**

3.1 Taylor's Theorem for a function of two variables.
3.2 Maclaurin's Theorem for a function of two variables.
3.3 Absolute and Relative Maxima and Minima.
3.4 Necessary Condition for extrema.
3.5 Critical Point, Saddle Point.
3.6 Sufficient Condition for extrema.
3.7 Lagrange's Method of undetermined multipliers.

Unit -4 : Double and Triple Integrals **Periods – 15, Marks – 15**

4.1 Curve Tracing
4.2 Double integrals by using Cartesian and Polar coordinates.
4.3 Change of Order of Integration.
4.4 Area by Double Integral.
4.5 Evaluation of Triple Integral as Repeated Integral.
4.6 Volume by Triple Integral.

Reference Books –

1. Mathematical Analysis: S.C. Malik and Savita Arora. Wiley Eastern Ltd, New Delhi.
2. Calculus of Several Variables: Schaum's outline Series.
3. Mathematical Analysis: T.M.Apostol. Narosa Publishing House, New Delhi, 1985
4. A Course of Mathematical Analysis: Shanti Narayan, S. Chand and Company, New Delhi.

MTH -232(A): Algebra

Unit-1 : Groups **Periods-15, Marks-15.**

1.1 Definition of a group.
1.2 Simple properties of group.
1.3 Abelian group.
1.4 Finite and infinite groups.
1.5 Order of a group.
1.6 Order of an element and its properties.

Unit-2 : Subgroups **Periods-15, Marks-15.**

2.1 Definition of subgroup, criteria for a subset to be a subgroup.
2.2 Cyclic groups.
2.3 Dihedral group (Definition and Examples only)
2.4 Coset decomposition.
2.5 Lagrange's theorem for finite group.
2.6 Euler's theorem and Fermat's theorem.

Unit -3 : Homomorphism and Isomorphism of Groups **Periods-15, Marks-15.**

3.1 Definition of Group Homomorphism and its Properties .
3.2 Kernel of Homomorphism and Properties.
3.3 Definition of Isomorphism and Automorphism of Groups.
3.4 Properties of Isomorphism of Groups.

Unit-4 : Rings **Periods-15, Marks-15.**

4.1 Definition and Simple Properties of a Ring.
4.2 Commutative Ring, Ring with unity, Boolean Ring.
4.3 Ring with zero divisors and without zero Divisors.
4.4 Integral Domain, Division Ring and Field. Simple Properties.

Reference Books –

1. Topics in Algebra: I. N. Herstein (John Wiley and Sons).
2. A first Course in Abstract Algebra: J. B. Fraleigh (Pearson).
3. University Algebra: N. S. Gopalakrishnan (New age international publishers).
4. A course in Abstract Algebra: Vijay K. Khanna and S.K.Bhambri (Vikas Publishing House Pvt, Ltd. Noida).

MTH -232(B): Theory of Groups

Unit-1 : Groups **Periods-15, Marks-15.**

1.1 Definition of a group.

1.2 Simple properties of group.

1.3 Abelian group.

1.4 Finite and infinite groups.

1.5 Order of a group.

1.6 Order of an element and its properties.

Unit-2 : Subgroups **Periods-15, Marks-15.**

2.1 Definition of subgroup, criteria for a subset to be a subgroup.

2.2 Cyclic groups.

2.3 Dihedral group (Definition and Examples only)

2.4 Coset decomposition.

2.5 Lagrange's theorem for finite group.

2.6 Euler's theorem and Fermat's theorem.

Unit -3 : Homomorphism and Isomorphism of Groups **Periods-15, Marks-15.**

3.1 Definition of group homomorphism and its properties .

3.2 Kernel of homomorphism and properties.

3.3 Definition of isomorphism and automorphism of groups.

3.4 Properties of isomorphism of groups.

Unit-4 : Group Codes **Periods -15, Marks 15.**

4.1 Message, Word,(m,n)- Encoding Function, Code Words.

4.2 Detection of k or fewer errors, Weight, Parity Check Code

4.3 Hamming Distance, Properties of the Distance Function, Minimum Distance of an encoding function.

4.4 Group Codes.

4.5 (n, m)- Decoding function, Maximum Likelihood Decoding Function.

4.6 Decoding procedure for a Group Code given by a Parity Check Matrix.

Reference Books –

1. Discrete Mathematical Structures: Bernard Kolman, Robert C. Busby and Ross (Prentice Hall of India New Delhi, Eastern Economy Edition).
2. Topics in Algebra: I.N. Herstein.
3. A first Course in Abstract Algebra: J. B. Fraleigh (Pearson).
4. University Algebra: N. S. Gopalakrishnan (New age international publishers).
5. A course in Abstract Algebra: Vijay K. Khanna and S.K.Bhambri (Vikas Publishing House Pvt, Ltd. Noida).

MTH- 233 : Practical Course based on MTH-231, MTH-232

Practical – 1 : Functions of Two or Three Variables

1. Evaluate limit if it exists for following functions.

   i) $\lim_{(x,y)\to(0\,,\,0)} \frac{\sin(x^2+y^2)}{x+y}, \quad x+y \neq 0$

   ii) $\lim_{(x,y)\to(0\,,\,0)} \frac{x^3y^3}{x^2+y^2}, \quad x^2+y^2 \neq 0.$

2. Let $f(x,y) = \frac{x^2y^2}{x^4+y^4-x^2y^2}$, $(x,y) \neq (0\,,\,0)$, verify that both the repeated limits exist and are equal but simultaneous limit does not exist.

3. i) Discuss the continuity of $f(x\,,\,y) = \begin{cases} \frac{2xy^2}{x^3+y^3}, & \text{if } (x\,,\,y) \neq (0\,,\,0) \\ 0, & \text{if } (x\,,\,y) = (0\,,\,0) \end{cases}$ at origin.

   ii) Show that the function $f(x\,,\,y) = \begin{cases} \frac{x^3-y^3}{x^2+y^2}, & \text{if } (x\,,\,y) \neq 0 \\ 0, & \text{if } (x\,,\,y) = (0\,,\,0) \end{cases}$

   is continuous at origin.

4. If $u = \log(\tan x + \tan y + \tan z)$, then prove that $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2.$

5. If $f(x\,,\,y) = \begin{cases} \frac{x^3y}{x^2+y^2}, & \text{if } x^2+y^2 \neq 0 \\ 0, & \text{if } (x\,,\,y) = (0\,,\,0) \end{cases}$, then show that $f_{xy}(0\,,\,0) \neq f_{yx}(0\,,0)$.

6. Show that the function $f(x,y) = \sqrt{|xy|}$ is continuous at (0 , 0) but not differentiable at (0 , 0).

7. Find approximately the value of $[(3.8)^2 + 2(2.1)^3]^{\frac{1}{5}}$.

8. Find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$, where $u = x+y+z$, $v = x+y$, $w = x-y-z$.

9. If $x = r\cos\theta$, $y = r\sin\theta$, Verify that $J.J' = 1$.

Practical - 2 : Composite functions and Mean Value Theorem

1. If $z = e^{xy^2}$, $x = t\cos t$, $y = t\sin t$, find $\frac{dz}{dt}$ at $t = \frac{\pi}{2}$.
2. If $w = f(x - y,\ y - z,\ z - x)$, find the value of $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$.
3. If z is function of x and y and if $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, then prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$.
4. If $z = f(x, y)$, where $u = 2x - 3y$, $v = x + 2y$, then prove that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 3\frac{\partial z}{\partial v} - \frac{\partial z}{\partial u}$.
5. If $u = f(e^{y-z},\ e^{z-x},\ e^{x-y})$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.
6. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, then show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin(1 - 4\sin^2 u)$.
7. If $u = x\phi\left(\frac{y}{x}\right) + \psi\left(\frac{y}{x}\right)$, then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$.
   Given that ϕ and ψ are at least twice differentiable.
8. If $f(x, y) = x^3 - xy^2$, show that θ used in the Mean value theorem applied to the points $(2, 1)$ and $(4, 1)$ satisfies the quadratic equation $3\theta^2 + 6\theta - 4 = 0$.
9. Let $f(x, y) = x^2y + 2xy^2$. Find the quadratic equation in θ by applying the Mean Value Theorem to line segment joining the points (1 , 2) to (3 , 3).

Practical – 3 : Taylor's Theorem and Extreme Values

1. Show that expansion of $\sin(xy)$ in powers of (x - 1) and (y - $\frac{\pi}{2}$) up to and including second term is
   $1 - \frac{\pi^2}{8}(x - 1)^2 - \frac{\pi}{2}(x - 1)(y - \frac{\pi}{2}) - \frac{1}{2}(y - \frac{\pi}{2})^2$.
2. Show that , for $0 < \theta < 1$
   $\sin x\ \sin y = xy - \frac{1}{6}[(x^3 + 3xy^2)(\cos\theta x \sin\theta y) + (y^3 + 3x^2y)\sin\theta x\cos\theta y]$.
3. Prove that $\sin(x + y) = (x + y) - \frac{(x+y)^3}{3!} + \cdots$
4. Expand $e^{2x}\cos y$ as Taylor's series about $(0, 0)$ up to first three terms.
5. Find the points (x , y) where the function u = xy (a - x - y) is maximum or minimum.
   What is the maximum value of the function?
6. Find the least value of the function $f(x, y) = xy + \frac{50}{x} + \frac{20}{y}$
7. Determine the minimum distance from the origin to the plane 3x + 2y + z -12 = 0.
8. Divide 24 in to three positive numbers such that their product is maximum.
9. Find the dimensions of a rectangular box open at the top whose volume is 108 cubic meters and its surface area is minimum.

Practical – 4 : Double and Triple integrals

1. Find the area bounded by the parabolas $y^2 = 2x$ and $x^2 = 2y$.
2. Evaluate $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}}\ dxdy$
3. Using triple integration, find the volume of the sphere of radius 5
4. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\ dxdydz$
5. Change the order of integration $\int_0^1 \int_{x^2}^{2-x} f(x,y)dxdy$

6. Change the order of integration $\int_0^3 \int_1^{\sqrt{4-y}} f(x,y)\ dxdy$
7. Evaluate $\int_{x=0}^1 \int_{y=0}^2 \int_{z=1}^2 x^2yzdzdydx$

8. Evaluate $\int_{y=0}^3 \int_{x=0}^2 \int_{z=0}^1 (x+y+z)dzdxdy$

9. Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=3,\ z=0$

Practical - 5 : Groups

1. Let $\mathbb{Q}^+$ denote the set of all positive rational numbers and for any a, b $\in \mathbb{Q}^+$ define $a*b = \frac{ab}{2}$. Show that $(\mathbb{Q}^+, *)$ is an abelian group.

2. Let G = { (a , b) : a , b$\in \mathbb{R}$, a $\neq$ 0}. Show that (G , Θ) is a non- abelian group, where (a , b) Θ (c , d) = (ac , ad + b).

3. Show that the set G of all 2x2 matrices of the form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ over $\mathbb{R}$ where ad – bc ≠ 0 forms a non abelian group under matrix multiplication.

4. Let G be a group in which $(ab)^n = a^nb^n$ for three consecutive integers n and for any a, b $\in$ G. Show that G is abelian.

5. Let G = $\left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} : a\ \in\ \mathbb{Q}^+ \right\}$. Prove that G is an abelian group with respect to multiplication of matrices.

6. (a) In the group $(\mathbb{Z}_8\ ,\ +_8)$, find (i) $\bar{3}^2$ ii) $\bar{3}^{-2}$ iii) $o(\bar{5})$ iv) $o(\bar{7})$

   (b) In the group $(\mathbb{Z}'_{11}\ ,\ \times_{11})$, find (i) $\bar{4}^3$ ii) $\bar{4}^{-3}$ iii) $o(\bar{9})$ iv) $o(\bar{7})$

7. If in a group G, the elements a and b commute, then prove that

   (i) a^{-1} and b^{-1} commute (ii) a and b^{-1} commute (iii) a^{-1} and b commute

Practical - 6: Subgroups

1. If H is a subgroup of a group G and $x \in G$, then show that $xHx^{-1} = \{ xhx^{-1} : h \in H\}$ is a subgroup of G.
2. Show that is $(\mathbb{Z}'_7 , \times_7)$ a cyclic group. Find all its generators, all its proper subgroups and order of every element.

3.(a) In a commutative group $(G , *)$, define $H = \{a \in G : a^k = e \text{ for some } k \in \mathbb{N} \}$. Determine whether $(H , *)$ is a subgroup of $(G , *)$.

(b) Let $H = \{ \overline{0} , \overline{2} , \overline{4} , \overline{6} \}$ be a subgroup of a group $G = (\mathbb{Z}_8 , +_8)$. Find all right(left) cosets of H in G.

4.Let A and B be two subgroups of a finite group G whose orders are relatively prime. Show that $A \cap B = \{e\}$.

5. (a) Show that every proper subgroup of a group of order 55 is cyclic.

(b) Find the remainder obtained when 15^{27} is divided by 8.

(c) Find the remainder when 41^{75} is divided by 3.

6. If G is a group of order 10, then show that it must have a subgroup of order 5.

Practical - 7 : Homomorphism and Isomorphism of Groups

1.Let $G = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a , b , c , d \in \mathbb{R}, \ ad - bc \neq 0 \right\}$, the group of all nonsingular matrices of order 2 over $\mathbb{R}$ under matrix multiplication and let $\mathbb{R}^* = \mathbb{R} - \{0\}$, the group of nonzero real numbers under multiplication. Define $f : G \to \mathbb{R}^*$ by $f(A) = |A|$, for all $A \in G$. Show that f is an onto group homomorphism and find its kernel.

2. If $G = \{1, -1, i, -i\}$ is a group under multiplication and $\overline{G} = \{ \overline{2} , \overline{4} , \overline{6} , \overline{8}\}$ is a group under multiplication modulo 10, then show that G and $\overline{G}$ are isomorphic.

3. (a) Let G be a group and $a \in G$. Show that $f_a : G \to G$ defined by $f_a(x) = axa^{-1}$, for all $x \in G$ is an automorphism.

(b) Show that the groups $G = \{1, -1, i, -i\}$ under usual multiplication and $\mathbb{Z}'_8 = \{ \overline{1} , \overline{3} , \overline{5} , \overline{7}\}$ under multiplication modulo 8 are not isomorphic.

4.Let G be a group and $f : G \to G$ be a map defined by $f(x) = x^{-1}$, for all $x \in G$. prove that
   a) if G is abelian, then f is an isomorphism.
   b) If f is a group homomorphism, then G is abelian.

5.Show that the set of all automorphisms of a group G forms a group under composition of mappings.

6.Let f and g be group homomorphisms from $G \to G$. Show that $H=\{x \in G : f(x) = g(x)\}$ is a subgroup of G.

Practical - 8(A) : Rings

1.(a) Show that $\mathbb{Z}_7 = \{ \overline{0} , \overline{1} , \overline{2} , \overline{3} , \overline{4} , \overline{5} , \overline{6} \}$ forms a ring under addition and multiplication modulo 7.

(b) In the ring $(\mathbb{Z}_{10} , +_{10} , \times_{10})$, find all divisors of zero.

2. Show that $\mathbb{Z}[i] = \{a + ib : a , b \in \mathbb{Z} \}$, the set of Gaussian integers, forms an integral domain under usual addition and multiplication of complex numbers.
3. Show that $R = \{a + b\sqrt{2} : a , b \in \mathbb{Z}\}$ is an integral domain under usual addition and multiplication.
4. In the ring $(\mathbb{Z}_7 , +_7 , \times_7)$, find

   (i) $-(\overline{4} \times_7 \overline{6})$ (ii) $\overline{3} \times_7 (\overline{-6})$ (iii) $(\overline{-5}) \times_7 (\overline{-5})$

   (iv) Units in $\mathbb{Z}_7$ (v) additive inverse of $\overline{6}$, (vi) zero divisors.

   Is $\mathbb{Z}_7$ a field or an integral domain? Justify.
5. Let $\mathbb{R}$ be the set of all real numbers. Show that $\mathbb{R} \times \mathbb{R}$ forms a field under addition and multiplication defined by $(a , b) + (c , d) = (a+c , b+d)$ & $(a , b) . (c , d) = (ac-bd , ad+bc)$.

Practical - 8 (B) : Group Codes

1. Consider the (3,8) encoding function $e : B^3 \to B^8$ defined by
   e(000) = 00000000, e(001) = 10111000, e(010) = 00101101, e(011) = 10010101,
   e(100) = 10100100, e(101) = 10001001, e(110) = 00011100, e(111) = 00110001
   (a) Find the minimum distance of e. (b) How many errors will e detect ?

2. Show that the (3,6) encoding function $e : B^3 \to B^6$ defined by
   e(000) = 000000 , e(001) = 001100, e(010) = 010011, e(011) = 011111,
   e(100) = 100101, e(101) = 101001, e(110) = 110110, e(111) = 111010
   is a group code. Also find the minimum distance of e.

3.Compute: (a) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \oplus \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

4.Let $H = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Determine the (2 , 5) group code $e_H : B^2 \to B^5$.

5.Consider the parity check matrix : $H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Decode the following words relative to a maximum likelihood decoding function associated with e_H :
a) 10100 b) 01101 c) 11011

6.Consider the parity check matrix : $H = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Determine the coset leaders for $N = e_H(B^3)$. Also compute the Syndrome for each coset leader and decode the code 001110 relative to maximum likelihood decoding function.

7.Let the (9,3) decoding function $d : B^9 \to B^3$ be defined by $d(y) = Z_1Z_2Z_3$, where
Z_i = 1, if $\{y_i, y_{i+3}, y_{i+6}\}$ has at least two 1's
= 0, if $\{y_i, y_{i+3}, y_{i+6}\}$ has less than two 1's, i = 1, 2, 3.
If $y \in B^9$, then determine d(y), where (i) y = 101111101 (ii) y = 100111100

SEMESTER – II

MTH -241 : Complex Variables

Unit-1 : Complex numbers **Period-15, Marks-15**

1.1 Complex numbers, modulus and amplitude, polar form
1.2 Triangle inequality and Argand's diagram
1.3 De Moivre's theorem for rational indices and applications
1.4 n^{th} roots of a complex number
1.5 Elementary functions (1) Trigonometric functions of a complex variable.
(2) Hyperbolic functions of a complex variable.

Unit-2: Functions of complex variables **Period-15, Marks-15**

2.1 Limits, Continuity, Derivative.
2.2 Analytic functions, Necessary and sufficient condition for analytic function.
2.3 Cauchy Riemann equations.
2.4 Laplace equations and Harmonic functions
2.5 Construction of analytic functions

Unit-3 : Complex integrations **Period-15, Marks-15**

3.1 Line integral and theorems on it.
3.2 Statement and verification of Cauchy-Gaursat's Theorem.
3.3 Cauchy's integral formulae for f(a), f'(a) and $f^n(a)$
3.4 Taylor's and Laurent's series.

Unit-4 : Calculus of Residues **Period-15, Marks-15**

4.1 Zeros and poles of a function.
4.2 Residue of a function
4.3 Cauchy's residue theorem
4.4 Evaluation of integrals by using Cauchy's residue theorem
4.5 Contour integrations of the type $\int_0^{2\pi} f(cos\theta,\, \sin\theta)d\theta \ \ and \ \ \int_{-\infty}^{\infty} f(x)dx$

Reference books –

1. Complex Variables and Applications ; R.V.Churchill and J.W. Brown.(McGraw-Hill)
2. Theory of Functions of Complex Variables : Shanti Narayan, S. Chand and Company, New Delhi.
3. Complex variables : Schaum's Outline Series.

MTH-242(A): Differential Equations

Unit-1 : Theory of ordinary differential equations **Period-15, Marks-10**

1.1 Lipschitz condition
1.2 Existence and uniqueness theorem
1.3 Linearly dependent and independent solutions
1.4 Wronskian definition
1.5 Linear combination of solutions
1.6 Theorems on i) Linear combination of solutions
ii) Linearly independent solutions
iii) Wronskian is zero
iv) Wronskian is non zero
1.7 Method of variation of parameters for second order L.D.E.

Unit-2 : Simultaneous Differential Equations **Period-15, Marks-10**

2.1 Simultaneous linear differential equations of first order
2.2 Simultaneous D.E. of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.
2.3 Rule I: Method of combinations
2.4 Rule II: Method of multipliers
2.5 Rule III: Properties of ratios
2.6 Rule IV: Miscellaneous

Unit-3 : Total Differential or Pfaffian Differential Equations **Period-15, Marks-10**

3.1 Pfaffian differential equations
3.2 Necessary and sufficient condition for integrability
3.3 Conditions for exactness
3.4 Method of solution by inspection
3.5 Solution of homogenous equation
3.6 Use of auxiliary equations

Unit-4 : Beta and Gamma Functions **Period-15, Marks-10**

4.1 Introduction
4.2 Euler's Integrals: Beta and Gamma functions
4.3 Properties of Gamma function
4.4 Transformation of Gamma function
4.5 Properties of Beta function
4.6 Transformation of Beta function
4.7 Duplication formula

Reference Book-

1. Ordinary and Partial Differential Equation by M. D. Rai Singhania, S. Chand & Co.
2 . Mathematical Analysis by S.C. Malik and Savita Arora.

MTH-242(B): Differential and Difference Equations

Unit-1 : Theory of ordinary differential equations **Period-15, Marks-10**

1.1 Lipschitz condition

1.2 Existence and uniqueness theorem

1.3 Linearly dependent and independent solutions

1.4 Wronskian definition

1.5 Linear combination of solutions

1.6 Theorems on i) Linear combination of solutions

ii) Linearly independent solutions

iii) Wronskian is zero

iv) Wronskian is non zero

1.7 Method of variation of parameters for second order L.D.E.

Unit-2 : Simultaneous Differential Equations **Period-15, Marks-10**

2.1 Simultaneous linear differential equations of first order

2.2 Simultaneous D.E. of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.

2.3 Rule I: Method of combinations

2.4 Rule II: Method of multipliers

2.5 Rule III: Properties of ratios

2.6 Rule IV: Miscellaneous

Unit-3 : Total Differential or Pfaffian Differential Equations **Period-15, Marks-10**

3.1 Pfaffian differential equations

3.2 Necessary and sufficient condition for integrability

3.3 Conditions for exactness

3.4 Method of solution by inspection

3.5 Solution of homogenous equation

3.6 Use of auxiliary equations

Unit-4 : Difference Equations **Period-15, Marks-10**

4.1 Introduction, Order of difference equation, degree of difference equations

4.2 Solution to difference equation and formation of difference equations

4.3 Linear difference equations ,Linear homogeneous difference equations with constant coefficients

4.4 Non homogenous linear difference equation with constant coefficients

Reference Book-

1. Ordinary and Partial Differential Equation by M. D. Rai Singhania, S. Chand & Co.
2. Numerical Methods by Dr.V. N. Vedamurthy and Dr. N. Ch. S. N. Iyengar

MTH- 243 : Practical Course based on MTH-241, MTH-242

Practical - 1 : Complex numbers

1. Determine the region in the Z-plane represented by $|z-3|+|z+3|=10$.

2. a) Using De Moivre's theorem prove

   i) $\cos 7\theta = \cos^7\theta - 21\cos^5\theta \cdot \sin^2\theta + 35\cos^2\theta \cdot \sin^4\theta - 7\cos\theta \cdot \cos^6\theta$

   ii) $\sin 7\theta = 7\cos^6\theta . \sin\theta - 35\cos^4\theta . \sin^3\theta + 21\cos^2\theta . \sin^5\theta - \sin^7\theta$

   b) Using De Moivre's theorem express $\cos^6\theta$ in terms of the cosines of multiple angle.

3. a) Find all values of $(1-i)^{\frac{2}{5}}$

   b) Solve the equation $x^8 - x^4 + 1 = 0$.

4. Find the real and imaginary parts of the following

   a) $\cos(x+iy)$, b) $\cosh(x+iy)$

5. If $\sin(a+ib) = x+iy$, *then show that* (a) $\frac{x^2}{\cosh^2 b} + \frac{y^2}{\sinh^2 b} = 1$ (b) $\frac{x^2}{\sin^2 a} - \frac{y^2}{\cos^2 a} = 1$

Practical - 2 : Functions of complex variable

1. a) Evaluate $\lim\limits_{z\to i} \frac{3z^4-2z^3+8z^2-2z+5}{z-i}$ b) Evaluate $\lim\limits_{z\to e^{i\pi/3}} \frac{(z-e^{\frac{i\pi}{3}})z}{z^3+1}$

2. Find $f(1+i)$, if f is continuous at $z=1+i$, where $f(z) = \frac{z^4+4}{z-(1+i)}$, $z \neq 1+i$

3. a) Find an analytic function $f(z) = u+iv$, *if* $v = e^{-y}\sin x$ *and* $f(0)=1$.

   b) Find an analytic function whose imaginary part is $v = 3 + x^2 - y^2 - \frac{y}{2(x^2+y^2)}$, by Milne Thomson method.

4. Show that $u = \frac{1}{2}\log(x^2+y^2)$ satisfies Laplace's equation. Find its harmonic conjugate.

Practical - 3 : Complex Integrations

1. Evaluate $\int_0^{3+i} z^2\, dz$

   i) along the line $x=3y$ ii) along the real axis to 3 and then vertically to $3+i$.

   iii) along the parabola $x=3y^2$.

2. Evaluate $\int_C \frac{z+6}{z^2-4}\, dz$, where C is the circle $|z|=1$ by Cauchy's Gourast theorem.

3. Evaluate $\int_C \frac{ze^z}{(z-1)^3}\, dz$, *where C is circle* $|z-1|=2$, by Cauchy's integral formula.

4. Expand $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ in power of z valid in the regions :

   i) $|z|<2$, ii) $2<|z|<3$, iii) $|z|>3$.

5. Find Laurent's series for $f(z) = \frac{3z-3}{(2z-3)(z-2)}$ valid for $\frac{1}{2} < |z-1| < 1$.

Practical - 4 : Calculus of Residues

1. If $f(z) = \frac{e^z}{z(z-1)^3}$, Find the sum residues of f(z) at all these poles.

2. Evaluate $\int_{|z|=2} \frac{dz}{z^3(z+4)}$ by Cauchy's residue theorem.

3. Evaluate $\int_C \frac{z^2-2z}{(z+1)^2(z^2+4)}\, dz$ by Cauchy's residue theorem where C is the rectangle formed by the lines $x = \pm 2 , y = \pm 3$

4. Use contour integration to evaluate $\int_0^{2\pi} \frac{1}{3+2\cos\theta}\, d\theta$

5. Use calculus of residue to find value of integral $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9}\, dx$

Practical - 5 : Theory of ordinary differential equations

1 .If f is defined on the rectangle È x È ≤ a , È Y È ≤ b show that the function

f(x,y) = x siny + ycosx satisfies the Lipschitz condition . Find the Lipschitz constant..

2 . Show that Y= 3 e^{2x} + e^{-2x} – 3x is the unique solution of the initial value problem Y' -4Y = 12 X where Y (0) = 4, y'(0) =1.

3 . Show that the functions 1+x, x^2, 1+2x are linearly independent.

4. Show that e^x sin x and e^x Cos x are linearly independent solutions of differential equations Y" -2Y' + 2Y = 0 .

5 . Examine whether e^{2x} and e^{3x} are linearly independent solutions of differential equation Y" -5Y'+6Y =0 or not ?

6. Solve by method of variation of parameters Y" +Y = x .

7. Solve by method of variation of parameters Y" + 9Y = Sec3x .

8. Solve by method of variation of parameters Y" - 3Y'+2Y = $\frac{e^x}{1+e^x}$.

Practical - 6 : Simultaneous Differential Equations

1. Solve $a)\ \frac{dx}{0} = \frac{dy}{-z} = \frac{dz}{y}$ $\quad b)\ \frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2y^2z^2}$ $\quad c)\ \frac{dx}{dt} - y = 1,\ \ \frac{dy}{dt} + x = 1$

2. Solve $a)\ \frac{dx}{y} = \frac{dy}{x} = \frac{dz}{xyz^2(x^2-y^2)}$ $\quad b)\ \frac{dx}{-xy^2} = \frac{dy}{y^3} = \frac{dz}{2xz}$

3. Solve $\frac{dx}{x+y} = \frac{dy}{x-y} = \frac{z.dz}{x^2+2xy-y^2}$

4. Solve $\frac{dx}{y^2(x-y)} = \frac{dy}{z^2(y-z)} = \frac{dz}{x^2(z-x)}$

5. Solve $\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$

6. Solve $\frac{dx}{\sin(x+y)} = \frac{dy}{\cos(x+y)} = \frac{dz}{z}$

7. Solve $\frac{dx}{zx-ay} = \frac{dy}{zy+ax} = \frac{dz}{z^2+a^2}$

Practical - 7 : Total (Pfaffian) Differential equations

1. a) Show that the equation $yz^2(x^2-yz)dx + zx^2(y^2-xz)dy + xy^2(z^2-xy)dz = 0$
   Is integrable. Is it exact ? Verify .
   b) Solve: $2xy\,dx - x^2dy + y^2z\,dz = 0$
2. Solve : $\frac{yz\,dx}{x^2+y^2} - \frac{xz\,dy}{x^2+y^2} - \tan^{-1}\left(\frac{y}{x}\right)\,dz = 0$
3. Solve the following by the method of homogeneous differential equations :
   a) $(y^2z - y^3 + x^2y)dx - (x^2z + x^3 - xy^2)dy + (x^2y - xy^2)dz = 0$
   b) $(y^2 + z^2 - x^2)dx - 2xy\,dy - 2xz\,dz = 0$
   c) $(2xz - yz)dx + (2yz - xz)dy - (x^2 - xy + y^2)dz = 0$
4. Solve the following by method of Auxiliary equations :
   a) $(2xz - yz)dx + (2yz - xz)dy - (x^2 - xy + y^2)dz = 0$
   b)$(y^2 + z^2 + yz)dx + (z^2 + x^2 + zx)dy + (x^2 + y^2 + xy)dz = 0$
   c) $z(z - y)dx + z(z + x)dy + x(x + y)dz = 0$

Practical - 8 (A) : Beta and Gamma Functions

1.Show that $\int_0^\infty x^{m-1}\cos(ax)\,dx = \frac{\Gamma m}{a^m}\cos\left(\frac{m\pi}{2}\right)$

2. a) Show that $\int_0^1 \left[log\left(\frac{1}{x}\right)\right]^{n-1} dx = \Gamma n$

   b) Evaluate $\int_0^1 (x\,logx)^4\,dx$

3. a) Evaluate $\int_0^\infty \frac{dx}{3^{4x^2}}$

   b) Show that $\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$

4. Show that $\int_0^{\pi/2} \sqrt{tan\theta}\,d\theta \quad \int_0^{\pi/2} \sqrt{cot\theta}\,d\theta = \frac{\pi^2}{2}$

5. Evaluate $\int_3^7 (x-3)^{1/4}\,(7-x)^{1/4}\,dx$

6.. Evaluate $\int_{-1}^1 (1+x)^m\,(1-x)^n\,dx$

Practical - 8 (B) : Difference Equations

1. Form the difference equation corresponding to the following general solution :
   a) $y = c_1.3^x + c_2.8^x$ b) $y = (c_1 + c_2n)(-2)^n$ c) $y = c_1x^2 + c_2x + c_3$
2. Show that $y_x = c_1 + c_2.2^x - x$ is a solution of difference equation $y_{x+2} - 3y_{x+1} + 2y_x = 1$
3. Solve the following difference equations :
   a) $y_{x+1} + 3y_x = 0,\ y_0 = 2$ b) $\Delta^3u_n - 5\Delta u_n + 4u_n = 0$
   c) $y_{x+1} = -y_x + 1,\ x = 0,1,2,\ldots\ .and\ y_0 = 1$ d) $y_{x+1} - 3y_x = 1$
4. Solve the following non-homogeneous linear difference equations :
   a) $y_{x+1} - 2y_x = x + 1$ b) $y_{x+2} - 4y_x = 9x^2$
   c) $y_{x+2} - 4y_{x+1} + 4y_x = 3^x + 2^x + 4$ d) $\Delta y_x + \Delta^2 y_x = sinx$
5. Formulate the Fibonnaci difference equation and solve it.

New Equivalences

The equivalences for old courses of S. Y. B. Sc. Mathematics are given as follows:

Sem	Old Course (June 2013)	New equivalent course (June 2016)
I	MTH : 231 – Advanced Calculus	MTH : 231 – Calculus of Several Variables
	MTH : 232(A) – Topics in Algebra	MTH : 232(A) – Algebra
	MTH : 232(B) – Computational Algebra	MTH : 232(B) – Theory of Groups
II	MTH : 241 – Complex Analysis	MTH : 241 – Complex Variables
	MTH : 242(A) – Topics in Differential Equations	MTH : 242(A) – Differential Equations
	MTH : 242(B) – Differential Equations and Numerical Methods	MTH : 242(B) – Differential and Difference Equations
	MTH : 203 – Practical Course based on MTH-231, MTH-232, MTH-241, MTH-242	MTH : 233 – Practical Course based on MTH-241, MTH-242 & MTH : 243 – Practical Course based on MTH-241, MTH-242